MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.
Tel: +254(0) 799 529 958, +254(0) 799 529 959, +254 (0)712 524 293
Website: www.must.ac.ke Email: info@mucst.ac.ke

UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF EDUCATION ARTS
AND
SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE AND BACHELOR OF SCIENCE IN STATISTICS

SMS 3251/ SMS 3476: TIME SERIES ANALYSIS/ TIME SERIES ANALYSIS

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define the following terms as used in time series analysis:
   i. Time series [2 Marks)
   ii. Correlogram [2 Marks)
   iii. Autocorrelation function [2 Marks)

b) Describe four objectives of time series analysis. [4 Marks]

c) Assuming a four yearly cycle, calculate the trend by method of moving averages from the following data. Show the trend short term fluctuations. [6 Marks]

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Production	21	22	23	25	24	22	25	26	27	26

d) Describe the four components of a time series data [4 Marks]

e) Explain any four areas where time series is applicable [4 Marks]

f) Let $X_t = \frac{1}{2}e_t - 2 + \frac{2}{9}e_{t-}1 + e_t$ be a time series process where e_t is a white noise process with variance σ^2.Obtain the autocorrelation function of X_t [6 Marks]

QUESTION TWO (20 MARKS)

a) A statistician is considering using the following process to model a seasonal data set:

$$(1 - B^3)(1 - (\alpha + \beta)B + \alpha\beta B^2)X_t = e_t$$

where B is the backshift operator and e_t is a white noise process with variance σ^2.

A seasonal difference series is defined as follows:

$$Y_t = X_t - X_{t-3}$$

i. Express the equation for the original process $X_t$in terms of the seasonal difference series, $Y_t$and the backwards shift operator B. [2 Marks]

ii. Determine the range of values of α and β for which the seasonal difference series,Y_t is stationary. [3 Marks]

Let γk and ρk denote the values at lag k of the autocovariance and autocorrelation functions, respectively, of the seasonal difference series,Y_t. The first Yule—Walker equation for Y_t may be written as follows:

$$1 - (\alpha + \beta)\rho 1 + \alpha\beta\rho_2 = \frac{\sigma^2}{\gamma_0}$$

iii. Write down the second and third Yule—Walker equations for $Y_t$in terms of ρ_1 and ρ_2 . [3 Marks]

The Actuary has observed the following sample autocorrelation values for the series Y_t: $\hat{\rho}_1 = 0.5$ and $\hat{\rho}_2 = 0.2$

iv. Estimate, using the equations in part (iii), the parameters α and β based on this information. [6 Marks]

[Hint: let $M = \alpha + \beta$ and $N = \alpha\beta$ use the formula for finding the roots of a quadratic equation.]

v. Determine the values of the one-step ahead and two-step ahead forecasts, $\hat{x}_{550}$ and $\hat{x}_{551}$, respectively, based on the parameters estimated in part (iv) and the observed values $x_1, x_2, \dots x_{549}$ of X_t. [6 Marks]

QUESTION THREE (20 MARKS)

a) Consider an AR(2) process given by $X_t = \frac{7}{20}X_{t-1} + \frac{3}{20}X_{t-2} + e_t$. Determine the autocorrelation function where e_t is a white noise. [10 Marks]

b) Fit a straight line trend by the method of least squares and estimate the number of tourists that would arrive monthly in the year 2000. [10 Marks]

Year	1990	1991	1992	1993	1994	1995	1996	1997
Tourists in millions	18	20	23	25	24	28	30	26

QUESTION FOUR (20 MARKS)

A zero-mean, first-order moving average process is defined by the following equation:

$$X_t = e_t + be_{t-1}$$

Where $e_t$is a sequence of independent and identically distributed $N(0, \sigma^2)$random variables.

i. Derive, in terms of b, the value of p that minimises:

$$E[(X_t - pX_{T-1})^2]$$ [8 Marks]

ii. Comment on your answer to part (i) in the case where $b = 1$. [4 Marks]

iii. Determine, the case where $b = 1$, the values of q and r that minimizes.

$E[(X_t - pX_{t-1} - rX_{t-2})^2]$ [8 Marks]

QUESTION FIVE (20 MARKS)

a) Consider the time series process, X_t, given by:

$$X_t = aX_{t-1} + \frac{1}{2}X_{t-2} + e_t + e_{t-1}$$

where e_t is a sequence of independent and identically distributed $N(0, \sigma^2)$random variables.

Determine the values of the parameters a and b such that Xt is:

(i.) stationary. [6 Marks]

(ii.) invertible. [3 Marks]

(iii.) I (1). [3 Marks]

b) Consider the following and it to fit trend using the method of semi-averages. [8 Marks]

Year	1960	1961	1962	1963	1964	1965	1966	1967
Production	54	58	59	67	65	68	72	69