MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya

Tel: +254(0) 799 529 958, +254(0) 799 529 959, + 254 (0) 712 524 293,

Website: info@must.ac.ke Email: info@must.ac.ke

University Examinations 2023/2024

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF SCIENCE IN EDUCATION, BACHELOR OF EDUCATION TECHNOLOGY IN ELECTRICAL AND ELECTRONIC ENGINEERING, BACHELOR OF EDUCATION TECHNOLOGY IN CIVIL ENGINEERING, BACHELOR OF EDUCATION TECHNOLOGY IN MECHANICAL ENGINEERING AND BACHELOR OF SCIENCE MATHS PHYSICS

SMA 3250: VECTOR ANALYSIS

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Show that $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (2 marks)

b) Test if the points $a(1,2,-5), b(3,0,1)$ and $c(4,-1,4)$ are collinear (3 marks)

c) Given that $\vec{r}_1 = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{r}_2 = 2\hat{i} - 4\hat{j} - 2\hat{k}$ *and* $\vec{r}_3 = -\hat{l} + 2\hat{j} + 2\hat{k}$, *find*

i. $\vec{r} + \vec{r}\,2\vec{r}$ (2 marks)

ii. $\vec{r}_3 x \vec{r}_1$ (2 marks)

iii. $|\vec{r}_2 + \vec{r}_3|$ (2 marks)

iv. $\vec{r}_2 \bullet \vec{r}_1$ (2 marks)

d) Given $\vec{R} = \hat{l}\sin t + \hat{j}\cos t + \hat{k}$ find

i. $\left|\frac{d_2\vec{r}}{dt^2}\right|$ (3 marks)

ii. The unit tangent vector (2 marks)

iii. $\int_0^{\pi}\vec{R}\,dt$ The unit targets vector (2 marks)

e) If $\theta(x,y,z)=3x^2y-y^3z^2$ find

i. $\vec{z}\theta$ at the point (1,2,1) (3 marks)

ii. The directional derivative at (1,2,1) in the direction of $\vec{A}=4\hat{l}-3\hat{j}+12\hat{k}$

(4 marks)

f) Find the equation of a plane with normal vector $\hat{n}<2,-2,5>$ given that the point (2,1,3) lies on this plane (3 marks)

QUESTION TWO (20 MARKS)

a) Find an expression for the magnitude of a vector having point $p(x,y,z)$ and terminal point $Q(x_2,y_2,z_2)$ (5 marks)

b) Given $\vec{r}_1=2\hat{l}-\hat{j}+\hat{k},\ \vec{r}_2=\hat{l}+3\hat{j}-2\hat{k},\ \vec{r}_3=-2\hat{l}+\hat{j}-3\hat{k}\ and\ \vec{r}_4=3\hat{l}-2\hat{j}+5\hat{k}$. Find scalars a, b and c such that

$\vec{r}_4=a\vec{r}_1+b\vec{r}_2+c\vec{r}_3$ (5 marks)

c) Given that $\vec{A}=2\hat{l}+2\hat{j}-\hat{k}\ and\ \vec{B}=6\hat{l}-3\hat{j}+2\hat{k}$

i. Find the angle between the two vectors (4 marks)

ii. Find pro $\hat{j}\vec{B}$ (2 marks)

iii. Find the area of a triangle whose adjacent sides are the vectors $\vec{A}\,and\,\vec{B}$

(4 marks)

QUESTION THREE (20 MARKS)

a) A straight line passes through the points $P(1,3,-2\ and\ Q(4,1,5)$. Find

i. The vector equation of line PQ (2 marks)

ii. The Cartesian symmetric of the line PQ (2 marks)

iii. The parametric equation of the line PQ (3 marks)

b) Find the a cute angle between the lines

$$\frac{x-1}{5}=\frac{y+2}{4}=\frac{z}{6}a;nd\ \frac{x+3}{2}=\frac{y-5}{7}=\frac{z+1}{-1}$$ (7 marks)

c) Find the directional derivative of $Q(x,y,z)=x^2yz+4xz^2\ at\ (1,-2,-1)$ in the direction of

$2\hat{i}-\hat{j}-2\hat{k}$ (6 marks)

QUESTION FOUR (20 MARKS)

a) If $Q=2x^3y^2z^4$ find $\underset{\sim}{\nabla}\underset{\sim}{\nabla}\phi=\underset{\sim}{\nabla}^2\phi$ (4 marks)

b) Prove that $\underset{\sim}{\nabla}.(\underset{\sim}{A}+\underset{\sim}{B})=\underset{\sim}{\nabla}.\underset{\sim}{A}+\underset{\sim}{\nabla}.\underset{\sim}{B}$ (5 marks)

c) Prove that $\underset{\sim}{\nabla}x(\underset{\sim}{\nabla}\phi)=0$ (5 marks)

d) The acceleration of a particle at any time ≥ 0 is given by

$\underset{\sim}{a}=\frac{d\underset{\sim}{v}}{dt}=12\cos\underset{\sim}{i}-8\sin 2t\ \underset{\sim}{j}+16t\underset{\sim}{k}$. Find

i. $\underset{\sim}{v}$ at any time (3 marks)

ii. $\underset{\sim}{r}$ at any time (3 marks)

QUESTION FIVE (20 MARKS)

a) Find the total work done in moving a particle in a force field given by

$\underset{\sim}{F}=3xy\underset{\sim}{i}-szj+10x\underset{\sim}{k}$ along the curve $x=t^2+1, y=2t^2, z=t^3$ from t=1 to t=2

(5 marks)

b) Evaluate $\iint_s \underset{\sim}{A}.\underset{\sim}{n}\,ds\ where\ \underset{\sim}{A}=z\underset{\sim}{i}+x\underset{\sim}{j}-3y^2z\underset{\sim}{k}\ and\ s$ is the surface of the cylinder

$x^2+y^2=16$ included in the first octant between $z=0\ and\ z=5$ (6 marks)

c) Verify Green's theorem in the Name $\oint_c (xy+y^2)dx+x^2dy$ where c is the closed curve of

the region bounded by $y=x\ and\ y=x^2$ (9 marks)