MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya

Tel: +254(0) 799 529 958, +254(0) 799 529 959, + 254 (0) 712 524 293,

Website: info@must.ac.ke Email: info@must.ac.ke

University Examinations 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE IN EDUCATION

SMA 3451: TOPOLOGY II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Define the terms:

i. A connected set (1 mark)

ii. A path in a topological space x (1 mark)

iii. Homotopic paths (2 marks)

iv. Cover of a subset A of a topological space x (2 marks)

b) Consider the topology $c = \{x, \phi, \{a,b,c\}, \{c,d,e\}\{cy\} on\ X = \{a,b,c,d,e\}$

Show that the subset $A = \{a,d,e\} of\ x$ is disconnected (6 marks)

c) i. State the finite intersection property of a topological space X (2 marks)

ii. Prove that the class of open intervals $A = \left\{(0,1), \left(0, \frac{1}{2}\right), \left(0, \frac{1}{3}\right), \left(0, \frac{1}{4}\right), ...\right\}$

statisties the finite intersection property (5 marks)

d) Prove that continuous images of arc wise connected sets are arc wise connected (5 marks)

e) i. Define a compact subset of a topological space X (2 marks)

ii. Given that $A = (0,1)$ is an open interval of the real line $\mathbb{R}$, prove that A is not compact in $\mathbb{R}$ (4 marks)

QUESTION TWO (20 MARKS)

a) i. Define a disconnection of a subset A of a topological space X (2 marks)

ii. Let GUH be a disconnection of a subset A of a topological space. Prove that $A \cap G$ and $A \cap H$ are separated sets (8 marks)

b) Given that A is a compact subset of a Hausdorff space X and suppose that $P \epsilon X \setminus A$. Show that there exist open sets G and H such that $P \epsilon G, ACH and\ G \cap H = \emptyset$ (5 marks)

QUESTION THREE (20 MARKS)

a) i. Distinguish between a sequentially compact subset of a topological space X and a countably compact subset of X (4 marks)

ii. Prove that a continuous image of a sequentially compact set is sequentially compact (5 marks)

b) Prove that every bounded closed interval $A = [a,b]$ is countably compact (8 marks)

QUESTION FOUR (20 MARKS)

a) Define the terms;

i. Product topology for topological spaces $x\ and\ y$ (3 marks)

ii. Basic neighbourhoods of $(x,y) \epsilon X x Y$ (2 marks)

b) Consider the topology $T = \{X, \phi, \{b\}, \{a, c\}\}$ on $X = \{a, b, c\}$ and let the topology $T^* = \{Y, \emptyset, \{V\}\}$ of $Y = \{u, v\}$

i. Determine the defining subbase of the product topology on XxY (10 marks)

ii. Determine the defining base for the product topology on XxY (5 marks)

QUESTION FIVE (20 MARKS)

a) Let X be a topological space. Show that the following conditions are equivalent

i. X is disconnected

ii. There exists a non-empty proper subset of X which is both open and closed

(7 marks)

b) Consider the intervals $A = (0,1), B = (1,2)\ and\ c = [2.8)$ of $\mathbb{R}$. Prove that

i. A and B are separated sets (4 marks)

ii. B and C are not separated sets (4 marks)

c) Show that a subset E of the real line $\mathbb{R}$ containing at least two points is connected if and only if E is an interval (5 marks)