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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED STATISTICS

SMS 5129: TIME SERIES ANALYSIS

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Define the following terms as used in time series analysis

i. white noise (1mark)

ii. causality (1mark)

iii. invertibility (1mark)

iv. stationarity (1mark)

(b) Briefly explain four characteristics of time series data [4 marks]

(c) Prove that a Random Walk has a weakly stationarity [5 marks]

(d) Consider the times sequence $X_t = A\sin(\varpi t + \theta)$ sin where A is a random variable with zero mean and a unit variance while θ is a random variable with uniform distribution in the interval $(\pi, -\pi)$, and independent of A. Show X_t that the process is covariance stationary. [6 marks]

(e) Consider the time series process, X_t, given by:

$$X_t = aX_{t-1} + \frac{1}{2}X_{t-2} + e_t + Be_{t-1}$$

where e_t is a sequence of independent and identically distributed $N(0, \sigma^2)$ random variables. Determine the values of the parameters a and b such that X_t is:

(i.) Stationary. [4 marks]

(ii.) Invertible. [3 marks]

(f) Explain the use Box-Jenkins Model in time series Analysis [4 marks]

QUESTION TWO (20 MARKS)

Let X_t be the process defined by:

$$X_t = \sum_{i=1}^{t} Y_1$$

where:

$Y_t = e_t + be_{t-1}$ and e_t is a sequence of independent and identically distributed $N(0, \sigma^2)$ random variables.

(i) State the values of p, d and q for which X_t is an $ARIMA\ (p, d, q)$ process. [3 marks]

(ii) Demonstrate that $Var(Y_t) = (1+b^2)\sigma^2$ and $Cov(Y_t, Y_{t-1}) = b\sigma^2$. [6 marks]

(iii) Demonstrate that

$Var(X_t) = t(1+b^2)\sigma^2 + 2(t-1)b\sigma^2$ and that

$Cov(X_t, X_{t-1}) = (t-k)(1+b^2)\sigma^2 + 2((t-k)-1)b\sigma^2$ for $0 < k < t$ [8 marks]

(iv) Explain what the results in part (iii) imply about the shape of the autocorrelation function of X_t. [3 marks]

QUESTION THREE (20 MARKS)

Consider the following time series model:

$$Y_t = 1 + 0.6Y_{t-1} + 1.6Y_{t-2} + \varepsilon_t$$

where et is a white noise process with variance σ^2

i. Determine whether Yt is stationary and identify it as an $ARMA\ (p, q)$ process. [4 marks]

ii. Calculate $E(Y_t)$ [4marks]

iii. Calculate for the first four lags:

- The autocorrelation values p_1, p_2, p_3, p_4 (8marks)
- The partial autocorrelation values $\psi_1, \psi_2, \psi_3, \psi_4$ (4marks)

QUESTION FOUR (20 MARKS)

a) Describe multivariate time series [4 marks].

b) An index of salaries, S_t, and an index of prices, P_t,. are modelled as being related to each other in the following way:

$$\nabla \ln S_t = \theta_s + \alpha_1 \nabla \ln S_{t-1} + \alpha_2 \nabla \ln P_{t-1} + eS,_t$$
$$\nabla \ln P_t = P_s + \beta_1 \nabla \ln S_{t-1} + \beta_2 \nabla \ln P_{t-1} + ep,_t$$

where $eS,_t$ and ep, t are two independent zero-mean white noise processes, with variances σ_1^2 and σ_2^2 respectively, and θ_s and θ_p are constants.

(i) Explain why the above model is written in terms of $\nabla \ln P$ and $\nabla \ln S$ instead of just P and S. [4 marks]

(ii) Comment on whether it is reasonable that ∇lnS_t should be affected by ∇lnP_{t-1} and that ∇lnP_t should be affected by ∇lnS_{t-1}. [4 marks]

(iii) Use matrix notation to express $(\nabla lnS_t, \nabla lnP_t)^T$ as a Vector Autoregression and identify the order p of the $VAR(p)$ process. [4 marks]

(iv) Suppose the parameters of the model have been estimated. Describe the use of sensitivity analysis in determining the validity of the model. [4 marks]

QUESTION FIVE (20 MARKS)

Let A sample of size n is taken from a process, X_t, which is believed to be an ARMA(1,1) process of the form $X_t = aX_{t-1} + e_t + be_{t-1}$

where $|a|, |b| < 1$. The sample autocorrelations at lag 1 and lag 2 are 0.65 and 0.325, respectively.

i. Estimate the parameters a and b by equating the sample autocorrelations to the theoretical values. [10 marks]

Fisher's transformation states that the sample correlation coefficient, r, between two random variables, Y and Z, is such that $\frac{1}{2} log\left(\frac{1+r}{1-r}\right)$ is approximately Normally distributed with mean $\frac{1}{2}\log\left(\frac{1+p}{1-p}\right)$ and variance $\frac{1}{n-3}$, where p is the theoretical correlation coefficient between Y and Z and n is the sample size.

ii. Determine the minimum value of n necessary to reject the null hypothesis that $b = 0$in favour of the alternative $b > 0$ at the 95% significance level. You should assume that a is equal to the value determined in part (i) and use Fisher's transformation on the autocorrelation at lag. [10 marks]