



MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS(STATISTICS), BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF EDUCATION ARTS

SMS 3353: TESTS OF HYPOTHESIS

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Distinguish between parametric and nonparametric tests and describe situations in which the use of nonparametric tests may be appropriate. (3 Marks)
- b) Explain and interpret the p-value as it relates to hypothesis testing. (2 Marks)
- c) Identify the appropriate test statistic and interpret the results for a hypothesis test concerning:
- The variance of a normally distributed population (2 Marks)
 - The equality of the variances of two normally distributed populations based on two independent random samples. (2 Marks)
- d) Independent random samples were selected from each of two normally distributed populations = 6 from population 1 and = 4 from population 2. The data is as shown:

Sample 1	3.1	4.3	1.2	1.7	0.7	3.4
Sample 2	2.3	1.4	3.7	8.9		



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Test the null hypothesis of $H_0: \sigma_1^2 = \sigma_2^2$ versus an alternate hypothesis of $H_1: \sigma_1^2 \neq \sigma_2^2$ use $\alpha = 0.01$. (6 Marks)

- e) Boxes of certain masses are normally distributed with mean μ and standard deviation 15 grams. A test of the null hypothesis $\mu = 375$ grams against the alternative $\mu > 375$ grams is carried out at 2.5% level using a random sample of 16 boxes.
- Show that the alternative hypothesis is accepted when $\bar{x} > 382.35$ grams where \bar{x} is the sample mean mass. (3 Marks)
 - Given that the actual value of $\mu = 385$ grams, find the probability of making a type two error and the power of the test. (3 Marks)
- f) Let X be $N(\mu; 4)$. We would like to test the null hypothesis Test $H_0: \mu = 10$ against $H_1: \mu > 10$. Let the critical region be defined by: $C = \{(x_1, x_2, \dots, x_9): \bar{X} \geq 11.5\}$ where X is the mean of a random sample of size 9 from the distribution.
- What is the significance level of this test? (4 Marks)
 - How is the power function $k(\mu)$ defined for the test? (3 Marks)
 - What are the values of $k(11)$ and $k(12.5)$? (2 Marks)

QUESTION TWO (20 MARKS)

- a) Explain a test statistic, Type I and Type II errors, a significance level, and how significance levels are used in hypothesis testing. (5 Marks)
- b) Given that X is a random variable which is normally distributed with unknown mean μ and random variance $\sigma^2 = 3$ and x_1, x_2, \dots, x_n is a random sample of size n drawn from X . Obtain the critical region for testing $H_0: \mu = \mu_0$ versus $H_1: \mu > \mu_0$. Take the level of significance to be α . (10 Marks)
- c) Suppose observations from a normal population with variance $\sigma^2 = 4$ are 6, 3, 1, 2, 4, 2, 7, 8, 3, 5, 4 and 9. Test the hypothesis $H_0: \mu = 2.4$ versus $H_1: \mu = 2.3$ Take the level of significance to be $\alpha = 0.05$ (5 Marks)

QUESTION THREE (20 MARKS)

- a) Let x_1, x_2, \dots, x_n random variables with the pdf $f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

Where $\lambda > 0$ is an unknown parameter. Determine the Uniformly Most Powerful size α test for testing $H_0: \lambda \geq \lambda_0$ versus $H_1: \lambda < \lambda_0$ where λ_0 is a specified value λ . Hence give, in terms of a statistic in which $f(x, \lambda)$ has a maximum likelihood ratio and the UMP size α test.

(10 Marks)

- b) Sixteen oil tins are taken at random from an automatic filling machine. The mean weight of the tins is 14.2kgs, with a standard deviation of 0.40kg. Can one conclude that the filling machine is wasting oil by filling more than the intended weight of 14kg at a significance level of 5%

(5 Marks)

- c) A manufacturer of golf balls claims that the company controls the weights of the golf balls accurately so that the variance of the weights is not more than $1mg^2$. A random sample of 31 golf balls yields a sample variance of $1.62 mg^2$. Is that sufficient evidence to reject the claim at $\alpha = 0.10$

(5 Marks)

QUESTION FOUR (20 MARKS)

- a) Explain a **decision rule**, the **power of a test**, and the **relation between confidence intervals and hypothesis tests**.
- b) If X is a normal random variable with mean 0, derive the uniformly most powerful test of $H_0: \sigma_1^2 = 2$ versus $H_1: \sigma_1^2 > 2$ of size α ; based on a random sample of n observations of X. if $n = 10$ and $\alpha = 0.05$, find the uniformly most powerful critical region.
- c) The quality Y of milk produced by a certain breed of cows was assumed to be roughly linearly related to the amount X of a certain food supplement put into the diet of the cows. Data was collected and are given below:

x: 0.11 0.20 0.33 0.60 0.12 0.16 0.20 0.33 0.61 0.70

y: 0.71 0.98 1.17 1.76 0.77 0.83 0.96 1.25 1.76 1.96

- i. Test $H_0: \beta = 0$ against $H_1: \beta \neq 0$ at $\alpha = 0.05$ (5 Marks)
ii. Test $H_0: \rho = 0$ against $H_1: \rho \neq 0$ at $\alpha = 0.10$ (6 Marks)



QUESTION FIVE (20 MARKS)

- a) Assume that X has a poisson distribution with mean θ : Consider testing $H_0: \theta = 1$ against $H_1: \theta = 2$ using the sequential probability ratio test. Find $E [N|H_0 \text{ is false}]$ if $\alpha = \beta = 0.05$:
(7 Marks)
- b) Obtain appropriate test statistic and interpret the results for a hypothesis test concerning the population mean of both large and small samples when the population is normally or approximately distributed and the variance is unknown. (8 Marks)
- c) The following information relates to the prices (in \$) of a product in two cities A and B.

CITY	A	B
MEAN PRICE	22	17
STANDARD DEVIATION	5	6

The observations relating to prices were made for 9 months in city A and 11 months in city B. Test at $\alpha = 0.01$ level whether there is any significant difference between the prices in the two cities assuming the variance is unknown. (5 Marks)

