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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMS 3453: STOCHASTIC PROCESS

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Differentiate the following

i. Stochastic model and stochastic process (2marks)

ii. Periodic and non-periodic marker chain (2marks)

b) i) Find the pgf of the Poisson distribution

$$F(x) = p(x) = \begin{cases} \frac{e^{-\lambda}\lambda^{e}}{x!} & , x = 0,1,\dots \\ 0 & , elsewhere \end{cases}$$ (2marks)

ii) using the pgf in (i) Find the mean and the variance of the distribution (5marks)

c) the transition probabilities of a marker chain are given in the following matrix

$$p = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

i) Find the conditional probability of moving from stat E_2 to E_1 (2marks)

ii) If the initial probability distribution is $a_1 = \Pr(E_1) = {}^1/_3, a_2 = {}^2/_3$ and $a_3 = 0$,. Find the probability that the process, is in E_3 at the third step starting from any state (2marks)

d) Identify all the irreducible closes set of marker chain with the transitional probabilities given below

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/4 & 1/4 & 0 & 1/2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(5marks)

e) 280 workers use a canteen everyday tea, coffees and Goroa are series as alternative with lunch or any working day, each worker choose anyone of the three beverages. The probability that a worker chooses the same beverage or change to another day is given by the following transitions

$$(E_1 = Tea, \quad E_2 = Coffee, E_3 = goroa)$$

$$p = \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix} \begin{bmatrix} E_1 & E_2 & E_3 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.3 & 0.1 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$

In the longrun hers many workers will take ah of the three beverages (5marks)

f) Show that a matrix $[\mathrm{I}_t - \theta]$ has an inverse and inverse matrix of $[\mathrm{I}_t - \theta]^{-1} = \mathrm{I}_t + \theta + \theta^2 - e^3 + \cdots$ (7marks)

g) Describe a simple random walk problem (2marks)

QUESTION TWO (20 MARKS)

a) Given a pgf of a positive value of x as $G(s) = \frac{2}{3-3s}$

b) Find the mean and variance of x (4marks)

c) If $p_n = p(x = n)_1$findp_0, p_1 and p_n in general (3marks)

d) If the sequence $\{a_n\}$ o given by the following formula $a_n = a_{n-1}\, n = 2\ and\ a_0 = 1, a_1 = 1)$. The number of terrific accidents that occur in a city within a month is a random variable with mean 8.4 and variance 2.3. the number of individuals injured in the different accidents. S is independently distributed with mean 3.0 and variance 1.8. find the expected number and variance of the total number of people injured in one given month in the city. (5marks)

QUESTION THREE (20 MARKS)

a) Consider a branching process where the probability distribution of the first generation (Z_n) is given by $p_0 = 1/8, p_1 = 3/5, p_2 = 3/8, p_3 = 1/8$ and $p_n = 0$ for $n > 3$ let z_n

be the size of n^{th} generation. Assuming that the process starts with a single individual. Find.

i. The mean and variance of the first generation (5marks)

ii. $E(z_3)$ and $Var(z_3)$ (4marks)

b) Probability that the process dies on or before the

i. 2nd generation (4marks)

ii. 3rd generation (4makrs)

iii. 4th generation (3marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms

i. Absorbing Marker Chain (2marks)

ii. Transient state (2makrs)

iii. Persistent state (2marks)

b) The transition probabilities of a marker chain are given the following matrix

c) $\left(\frac{1}{2}\ 0\ 0\ \frac{3}{8}\right) p = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

d) Show that the marker chain is egordic. (4makrs)

e) Find the invariant distribution and give the interpretation, hence or otherwise obtain the recurrence time of all the states (10marks)

QUESTION FIVE (20MARKS)

a) Find the formula for the convolution of the following pair of sequence

$a_n = 1, 0 \leq n \leq 4, a_n = 0$ for n>5 $b_0 = n$ for all n (6marks)

b) Solve the following recurrence relation $a_{n-1} - 2a_n = 2^n, n \geq 0, a_0 = 1$ (10marks)

c) Describe the following

i. Generating function (2marks)

ii. Branching process (2marks)