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UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR
OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3363: RISK THEORY FOR ACTUARIAL SCIENCE

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Define risk theory [2 Marks]
- b) State the two conditions that must hold for a risk to be insurable. [4 Marks]
- c) List five other risk criteria that would be considered desirable by a general insurer. [4 Marks]
- d) The random variable S has a compound Poisson distribution with Poisson parameter 4. The individual claim amounts are either 1, with probability 0.3, or 3, with probability 0.7. Calculate the probability that $S = 4$. [4 Marks]
- e) A compound random variable $S = X_1 + X_2 + \dots + X_N$ has claim number distribution:
$$P(N = n) = 9(n + 1)4^{n-2}, \quad n = 0, 1, 2, \dots$$
The individual claim size random variable, X , is exponentially distributed with mean 2. Calculate $E(S)$ and $var(S)$. [6 Marks]
- f) An insurer calculates the annual premiums for fire insurance of flats by increasing the risk premium by 30% and adding a £30 loading. The claim frequency is 3% and individual claim amounts can be assumed, to be:
- £2,000 with probability 0.9



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- £15,000 with probability 0.1.

Calculate the insurer's adjustment coefficient for these policies, to 2 significant figures.

[5 Marks]

- g) Individual claim amounts in a Poisson claims process with a frequency of 50 claims per year for the whole portfolio have mean £5,000 and standard deviation £2,500. If the annual premium rate is £300,000, calculate the tightest upper bound for the adjustment coefficient. [5 Marks]

QUESTION TWO (20 MARKS)

The aggregate claim amount from a portfolio has a compound negative binomial distribution.

- Show that if $S = X_1 + X_2 + \dots + X_N$, then; $M_S(t) = M_N[\log M_X(t)]$. [4 Marks]
- If N is Type 2 negative binomial distribution with $k = 2$ and $p = 0.9$, and X has a gamma distribution with $\alpha = 10$ and $\lambda = 0.1$, find $M_S(t)$. [6 Marks]
- Find the mean and variance of S . Using a suitable approximation, estimate the aggregate amount which will be exceeded with probability 0.1%. [4 Marks]
- The insurer in fact has 100 identical independent portfolios of this type. If:

$$T = S_1 + S_2 + \dots + S_{100}$$

find the moment generating function for T , and, using a normal approximation, estimate the total aggregate claim amount from the whole business which will be exceeded with probability 0.1%. Comment on your answers to parts (iii) and (iv). [6 Marks]

QUESTION THREE (20 MARKS)

A surplus process for an insurer is denoted as $U(t)$ with initial surplus U .

- Define the surplus process for this insurer in terms of the initial surplus, premium income and aggregate claims process. All notations used should be defined. [2 Marks]

An insurer writes a 1-year pet insurance policy. The policy covers 100 pets and will pay out \$10 on each death of an insured pet during the policy term. The pet mortality rate is assumed to be 2.5% per month with claims being independent and assumed to follow a Poisson process. Premiums are paid continuously at \$29 per month. The insurer has an initial surplus of \$50.

- Calculate the probability that the time to the first claim is longer than 1 month. [3 Marks]
- Calculate the expected value of $U(t)$ at:
 - the end of month 1. [3 Marks]



(b) the end of the year. [3 Marks]

iv. Calculate the probability that the initial surplus and the insurer's premium income will be insufficient to cover the total claims in the first year. [4 Marks]

The insurer increases the estimate of pet mortality to 3.25% per month.

v. Comment on the effect that this increase in the mortality estimate will have on the:

a) probability of ruin in the next year. [2 Marks]

b) ultimate probability of ruin. [3 Marks]

QUESTION FOUR (20 MARKS)

a) State three statistical distribution used to model aggregate claims [6 Marks]

b) An insurer assumes that the number of claims it incurs follows a Poisson process with parameter μ . The adjustment coefficient, R , for this insurer is a risk measurement parameter associated with its surplus process. It is the unique positive root of the following equation:
 $\mu M_X(R) = \mu + cR$ where $M_X(R)$ is the moment generating function of the individual claim amount distribution, X .

i. State what quantity is represented by c . [2 Marks]

The insurer assumes that claim amounts follow the gamma distribution with parameters $\alpha = 2$ and $\lambda = 1$.

ii. Write down the moment generating function of this gamma distribution. [2 Marks]

iii. Show that R satisfies the following equation: $cR^2 + R(\mu - 2c) + (c - 2\mu) = 0$. [7 Marks]

The insurer sets $\mu = 3.5$ and $c = 1$

iv. Solve the equation in part (iii) to determine the adjustment coefficient. [3 Marks]

QUESTION FIVE (20 MARKS)

A singer has the following utility function:

$$U(w) = \ln(w), w > 0$$

i. Show that this utility function is iso-elastic. [4 Marks]

The singer measures their wealth as the present value of their future earnings. They are concerned about possible future damage to their voice and consider the following scenarios:



Scenario	Probability (%)	Present value of earnings (\$)
No damage	94.9%	1,500,000
Slight damage	5.0%	1,300,000
Severe damage	0.1%	10,000

The singer is considering buying insurance to cover their loss of earnings if their voice is damaged. The insurance will pay a lump sum equal to the reduction in the present value of earnings compared to the 'no damage' scenario.

- ii. Calculate the singer's expected loss when compared to no damage. [2 Marks]
- iii. Calculate the maximum premium the singer would be willing to pay for this insurance based on their utility function. [4 Marks]
- iv. Discuss briefly what your answers to parts (ii) and (iii) suggest about the singer's appetite to risk. [4 Marks]
- v. Suggest two reasons why an insurer may not want to cover this risk. [6 Marks]