



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## University Examinations 2023/2024

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR  
OF MATHEMATICS AND COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN  
MATHEMATICS

### SMA 3353: RING THEORY

DATE: APRIL 2024

TIME: 2 HOURS

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INSTRUCTIONS: Answer question *one* and any other *two* questions

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#### QUESTION ONE (30 MARKS)

- a) Explain the following terms as used in ring theory
- i. An integral ring (1 mark)
  - ii. Division ring (1 mark)
  - iii. Left ideal of a ring (2 marks)
  - iv. Homomorphism of a ring (2 marks)
- b) Prove that the intersection of two subrings of a ring  $R$  is a subring of  $R$  (5 marks)
- c) Prove that the set  $S = \{a + b\sqrt{3} \mid a, b \in \mathbb{Z}\}$  is a subring of the ring  $\mathbb{R}$  of real numbers (6 marks)
- d) Show that the polynomial  $f(x) = x^3 + x + 1$  is irreducible over the field  $F$  of integers modulo 5, ie  $(F, +_5, \cdot_5)$  (6 marks)

- e) Let  $(\mathbb{Z}, +, \cdot)$  be a ring of integers and  $(5\mathbb{Z}, +, \cdot)$  be an ideal generated by 5. Prove that the system  $(\mathbb{Z}/5\mathbb{Z}, +, \cdot)$  is a quotient ring (7 marks)

**QUESTION TWO (20 MARKS)**

- a) Prove that in a ring of all  $2 \times 2$  matrices over integers, the set  $u = \left\{ \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}$  is a left ideal but not a right ideal (7 marks)
- b) Prove that the intersection of two ideals of a ring R is an ideal of R (8 marks)
- c) Using the ring  $(\mathbb{Q}, +, \cdot)$  of rational numbers and the subring  $(\mathbb{Z}, +, \cdot)$  of  $(\mathbb{Q}, +, \cdot)$  show that a subring is not necessarily an ideal of the ring (5 marks)

**QUESTION THREE (20 MARK)**

- a) State the division algorithm of polynomials (3 marks)
- b) Given that  $f(x) = 6x^3 - 4x^2 + 8$  and  $g(x) = 2x^2 - 3x + 2$ . Find polynomials  $q(x)$  and  $r(x)$  such that  $f(x) = q(x).g(x) + r(x)$  (5 marks)
- c) Find the G.C.D of  $f(x) = x^3 - 2x^2 + 3x - 7$  and  $g(x) = x^2 + 2$  and express it in the form  $d(x) = m(x).f(x) + n(x).g(x)$  (8 marks)
- d) Determine whether the polynomial  $f(x) = 1 + x + x^3 + x^4$  is reducible over a field F of  $\mathbb{R}$  (4 marks)

**QUESTION FOUR (20 MARKS)**

- a) Define a left coset of a ring  $\mathbb{R}$  (3 marks)
- b) Let  $(I, +, \cdot)$  be an ideal of the ring  $(R, +, \cdot)$  prove that the system  $(R/I, +, \cdot)$  is a quotient ring of R by I (10 marks)

- c) Prove that an ideal  $(P, +, \cdot)$  of a commutative ring  $(R, +, \cdot)$  is prime if and only if  $(R/P, +, \cdot)$  is an integral domain (7 marks)

**QUESTION FIVE (20 MARKS)**

- a) i. Define the term unit group of a ring  $(R, +, \cdot)$  (2 marks)  
ii. Determine the unit group of the ring  $(\mathbb{Z}, +, \cdot)$  of integers (3 marks)
- b) Prove that  $\varphi: Q \rightarrow M_3(Q)$  defined by  $\varphi(u) = \begin{bmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & u \end{bmatrix}$  is a ring homomorphism (10 marks)
- c) Let  $n \in \mathbb{Z}$  be a positive integer. Prove that  $\psi: \mathbb{Z} \rightarrow \mathbb{Z}$ , defined by  $\psi(a) = n a$  is not a ring homomorphism (5 marks)