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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5138: RIEMANNIAH GEOMETRY

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: Answer any THREE questions.

QUESTION ONE (20 MARKS)

a) Write the law of transformation for the tensor B_{ijk}^{mn} (4marks)

b) Find the unit vectors e_r, e_θ, e_ϕ of spherical coordinates system in terms of i,j,k(8marks)

c) Prove that a cylindrical coordinate system is orthogonal (8marks)

QUESTION TWO (20 MARKS)

a) Define the following terms;

i. Hausdorff space (1mark)

ii. Symmetric tensor (2marks)

iii. Skew symmetric tensor (2marks)

b) Given that A_r^{pq} and B_r^{pq} are tensors show that their sum and difference are tensors (6marks)

c) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical coordinates (9marks)

QUESTION THREE (20 MARKS)

a) Express the vector $\vec{A} = 2yi - zj + 3xk$ in spherical coordinates and determine Ar, Aθ and A$\emptyset$. (8marks)

b) Determine the square of the element of are length in spherical forms coordinate form (8marks)

c) Prove that $\frac{d}{dt}(e_e) = \Phi e_\phi$ where the dot denotes differentiation w.r.t.t (4marks)

QUESTION FOUR (20 MARKS)

a) Show that the velocity of a fluid at any point is a contravariant tensor of rank one (5marks)

b) Prove that the contraction of the tensor A_q^p is invariant (scalar) (7marks)

c) Given the tensors A_r^p and B_t^{qs}, prove that their inner product in a tensor of rank three (8marks)

QUESTION FIVE (20MARKS)

a) A quantity A(j,k,l,m) which is a function of coordinate x' transforms to another coordinate system $\overline{x}^{-l}$ according to the rule;

$$\overline{A}(p,q,r,s) = \frac{\partial x^j}{\partial x^{-p}} \frac{\partial x^{-q}}{dx^k} \frac{\partial x^{-r}}{\partial x^l} \frac{\partial x^{-5}}{\partial x^m} \; A(j,k,l,m)$$

i. Determine whether the quantity is a tensor (5marks)

ii. If the quantity is a tensor, write in suitable notation (2marks)

iii. Give the contravariant, covariant order and rank (4marks)

b) Given that a tensor A_{st}^{pqr} is skew symmetric with respect to indices p and q in one coordinate system, show that it remains skew symmetric with respect to p and q in any other coordinate system (3marks)

c) Determine the metric tensor in cylindrical coordinates (6marks)