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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

SMS 3457: REGRESSION MODELING II

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Explain the following as used in modeling; (6 marks)

i. intrisically linear model

ii. factor loading

iii. homoscedasticity

(b) Given a simple linear regression model, such that the error terms are identically and independently distributed with mean 0 and constant finite variance, σ^2, show that $\text{var}(Y_i / X_i) = \sigma^2$ (5 marks)

(c) Giving relevant examples, discern between factor analysis and principal component analysis. (5 marks)

(d) Given the error sum of squares for any model is given by SSE = $\sum_{i=1}^{n} \varepsilon = \sum_{i=1}^{n} (y_i - \hat{y}i)^2$, show that $\hat{\sigma}^2 = \frac{E(SSE)}{(n-2)}$. (6 marks).

(e) Describe the departure of k nearest neighbor from kernels as used in simple nonparametric regression. (4 marks)

(f) Apart from homoscedasticity, explain any two diagnostic tests in regression analysis. (4 marks)

QUESTION TWO (20 MARKS)

(a) Given the model $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ with $\varepsilon_i \sim NIID(0, \sigma^2)$ show that $\hat{\beta}_1$ is an unbiased estimator for β_1 (4 marks)

(b) Giving relevant examples, describe a non-linear regression model. (3 marks)

(c) Given that $f(X, \gamma) = p_0 + p_1[\exp(p_2 Y)]$ is intrinsically linear, write it in the form $g(X, \gamma) = \alpha + \beta Y + \varepsilon$ (4 marks)

(d) Let $X_i = r_0 e^{\gamma_1 Y_I} + \varepsilon$ be a two-parameter non-linear exponential model. Using the method of ordinary least squares, derive the two normal equations for the least squares estimates. (9 marks)

QUESTION THREE (20 MARKS)

(a) Explain two advantages of discriminant analysis as used in data reduction.

(3 marks)

(b) Let be a random variable with the distributions

$$\Pi_1 : f_1(y) = p(y=0) = p(y=1) = \frac{1}{2}$$

$$\Pi_2 : f_2(y) = p(y=0) = \frac{1}{4}; p(y=1) = \frac{3}{4},$$

construct the classification procedure

(6marks)

(c) Let X $\sim N(\mu, \Sigma)$ be tri-variate normal random vector with $\sum = \begin{pmatrix} 8 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. Find;

i. the second principal component. (7 marks)

ii. the proportion of variance explained by the first principal component.

(4 marks)

QUESTION FOUR (20 MARKS)

(a) Giving real life examples, explain a simple linear regression model. (3 marks)

(b) Briefly discuss a neural network as used in regression modeling (4 marks)

(c) A regression model is to be developed for predicting the ability of soil to absorb chemical contaminants. Ten observations have been taken on a soil absorption index (y) and two regressors: $x_{1=}$amount of extractable iron ore and $x_{2=}$amount of bauxite. We wish to fit the model $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$. Some necessary quantities are:

$$X^1 X = \begin{bmatrix} 24..8281 & & \\ 204.7873 & 2388.2627 & \\ 8221.6820 & 76669.7725 & 3501398.349 \end{bmatrix} and X^1 Y = \begin{bmatrix} 725.82 \\ 8,008.37 \\ 274,811.31 \end{bmatrix}$$

i. Estimate the regression coefficients of the model specified above. (6 marks)

ii. Predict the soil absorption ability if $x_{11} = 3$ and $x_{12} = 50$ (4 marks)

iii. If the observed value y_1 =9.95, calculate its corresponding residual. (3 marks)

QUESTION FIVE (20 MARKS)

(a) Highlight the motivations of non-parametric regression. (4 marks)

(b) Let $\{X_i, Y_i\}_{i=1}^{5}$ be $\{(2,6),7,10),(4,8),(3,4),(6,12)\}$, compute the k-NN estimate $\hat{m}_3(4)$. (3 marks)

(c) Briefly explain training as used in machine learning. (3 marks)

(d) Let $X^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, X^{(2)} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, X^{(3)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, X^{(4)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ with correct classification $Z^{(1)} = Z^{(2)} = Z^{(3)} = 1, Z^{(4)} = 0$. The perceptron with weight w_0, w_1, w_2 classifies an object as 1 if and only if $Y = w^T X = w_0 X_0 + w_1 X_1 + w_2 X_2 > 0$ and 0 elsewhere. As initial vector, use $s = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and set $\eta = 1$. (10marks)