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University Examinations 2023/2024

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN EDUCATION, BACHELOR OF MATHEMATICS AND COMPUTER AND BACHELOR OF SCIENCE IN MATHEMATICS

SMA 3351: REAL ANALYSIS II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Differentiate between the term partition and norm of the partition (4 marks)

b) Let $P_1 = \{1,2,4,8,9,6,4,3,2\}$ *and* $P_2 = \{1,3,7,4,8,10,9\}$. Find the common refinement of P_1 and P_2 (2 marks)

c) If $f(x) = 2x - 1 \, on \, [0,1] \, and \, p = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ .Find $u(p, f) \, and \, L(p, f)$ (4 marks)

d) Evaluate $\int_0^{\pi} x d(\cos x)$ (5 marks)

e) Given the power series $\sum_{n=1}^{\infty} \frac{(4x+3)^n}{n^3}$ find the radius of convergence and interval of the regions of convergence (5 marks)

f) Show that $f(x) = \sin x$ is of bounded variations over a finite interval $[a, b]$ (5 marks)

g) Prove that if $\sum_{n=1}^{\infty} z_n$ is absolutely convergent, then $\sum_{n=1}^{\infty} z_n$ is convergent but the converse is not necessarily true (5 marks)

QUESTION TWO (20 MARKS)

a) Let $f(x)=\begin{cases} 0\,if\;\; x\,is\,rational \\ 1\,if\;\; x\,is\,irrational \end{cases}$. examine whether $f(x)$ is Riemann integrable on $[0,1]$

(5 marks)

b) Prove that any function of bounded variations is necessarily bounded (6 marks)

c) Show that $f(x)=x^2+1$ is Riemann integrable in $[0,1]$. Hence or otherwise find $\int_0^1 f(x)dx$

(9 marks)

QUESTION THREE (20 MARK)

a) Consider the function defined by $f(x)=\begin{cases} x^2\cos\frac{1}{2} & x\neq 0 \\ 0 & x=0 \end{cases}$. Examine whether $f(x)$ is of bounded variation on $[0,1]$ (3 marks)

b) Prove that if f maps $[a,b]$ to $\mathbb{R}$ and is continuous on interval $[a,b]$ and its derivative exist and bounded. Then $f \in BV(a,b)$ (5 marks)

c) Prove that if f and g are functions of bounded variations and c is a constant, then $f.g$ is of bounded variation (6 marks)

d) Evaluate $\int_{-1}^{1} xd(e^{|x|})$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Define the term function of bounded variations (3 marks)

b) Prove that $E(x).E(y) = E(x+y)$ (5 marks)

c) If $f(x) = c$ is a constant function, then prove that $f \epsilon \mathbb{R}[a, b]$ and

$\int_a^b f(x)dx = \int_a^b cdx = c(b-a)$ (5 marks)

d) Given the power series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n3^n}$. Find the radius of convergence and interval of the region of convergence (7 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\int_0^{\pi} xd(\sin x)$ (5 marks)

b) State and prove criteria for Riemann integrability (7 marks)

c) Let $f:[a,b] \to \mathbb{R}$ be a uniformly continuous function on $[a,b]$ the f is Riemann integrable. Prove (8 marks)