MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya

Tel: +254(0) 799 529 958, +254(0) 799 529 959, + 254 (0) 712 524 293,

Website: info@must.ac.ke Email: info@must.ac.ke

University Examinations 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

THIRD YEAR SECOND SEMESTER BACHELOR OF SCIENCE DATA SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER

SMA 3301: REAL ANAYSIS I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Let S be a non-empty subset of $\mathbb{R}$. Define the terms upper bound, lower bound, supremum and infimum of S (4 marks)

b) Find the supremum, infimum, maximum and minimum of the following sets if they exists

i. $s_1 = \left\{(-1)^n(1+\frac{1}{n}) : n \in \mathrm{N}\right\}$ (2 marks)

ii. $s_2 = \left\{r \in Q : -1 \le r \le \sqrt{2}\right\}$ (2 marks)

c) i. Define an open set (2 marks)

ii. Prove that the intersection of a finite collection of open sets is open (5 marks)

d) State the definition of convergence of a sequence of real number $(a_n), n\epsilon\mathbb{N}$ to a limit $a\epsilon\mathbb{R}$, and use this definition to show that $\frac{2n}{n^2+1} \to 1 as\, n \to \infty$ (7 marks)

e) i. Define conditional convergence of a sequence in $\mathbb{R}$ (4 marks)

ii. Show that the series $\sum_{n=1}^{\infty}(-1)^{n+1}(\frac{1}{n})$ is conditionally convergent (6 marks)

QUESTION TWO (20 MARKS)

a) i. Define continuity of a function $f : S \to \mathbb{R}$ (3 marks)

ii. Show that the function $f(x) = \begin{cases} \frac{x^2-4}{x-2}, x \neq 2 \\ 4 \quad, x = 2 \end{cases}$ is continuous at $x = 2$ (4 marks)

b) i. Show that the function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, x \neq 1 \\ 5 \quad, x = 1 \end{cases}$ is discontinuous at $x = 1$

(4 marks)

ii. How should $f(x)$ in b(i) above be defined to make it continuous at $x = 1$?

(3 marks)

c) i) Define uniform continuity of a function $f : s \to \mathbb{R}$ (2 marks)

ii) Prove that the function $f(x) = 2x$ is uniformly continuous on $\mathbb{R}$ (4 marks)

QUESTION THREE (20 MARK)

a) i. Define a Cauchy sequence (2 marks)

ii. Show that the sequence $\left(\frac{1}{2^n}\right)$ is a Cauchy sequence (6 marks)

b) i. Define the terms limit superior and limit inferior (2 marks)

ii. Prove that a sequence (x_n) of real numbers converges if and only if limit superior and limit inferior are equal (6 marks)

c) let $x = (0,1] \subset \mathbb{R}$. Using the sequence $x_n = \left(\frac{1}{n}\right)$ in X show that not every Cauchy sequence is convergent (4 marks)

QUESTION FOUR (20 MARKS)

a) i. Define a closed set (2 marks)

ii. Prove that a set $S \subset \mathbb{R}$ is closed if and only if s^c is open (6 marks)

b) Prove that the union of any finite number of closed sets is closed (5 marks)

c) i. Define a compact set in $\mathbb{R}$ (3 marks)

ii. let $H = [0, \infty]$. Show that H is not compact (4 marks)

QUESTION FIVE (20 MARKS)

a) Define the following

i. Even number (1 mark)

ii. Rational number (2 marks)

iii. Interior point of a set S (2 marks)

b) i. State the completeness axiom of the set $\mathbb{R}$ of real numbers (2 marks)

c) Prove that $\sqrt{3}$ is irrational (5 marks)

d) Given that a and b are rationals with $b \neq 0$ and S is an irrational number such that $a - bs = t$. Prove that t is irrational. Hence show that $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ is irrational (8 marks)