MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.
Tel: +254(0) 799 529 958, +254(0) 799 529 959, +254 (0)712 524 293
Website: www.must.ac.ke Email: info@mucst.ac.ke

UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS, BACHELOR OF SCIENCE (MATHEMATICS AND PHYSICS) AND BACHELOR OF SCIENCE (PHYSICS)

SPH 3350: QUANTUM MECHANICS I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Explain the meaning of the following terms as used in quantum mechanics

i. Coherent states (2 Marks)

ii. Scattering states (2 Marks)

iii. Bound stales (2 Marks)

iv. Tunneling (2 Marks)

v. Stationary states

b) In quantum mechanics every observable corresponds to a symmetric or Hermitian operator. Show that (2 Marks)

i. Eigenvalues corresponding to the Hermitian operators are real (5 Marks)

ii. Eigenstates corresponding to distinct eigenvalues are orthogonal (5 Marks)

c) i. If ψ, Θ and x are wave functions in the Hilbert space H, and $\lambda, \mu\ \theta.$ are complex numbers, prove that $< \psi|\lambda\theta + \mu\chi >= \lambda < \psi|\theta > +\mu < \psi.|\chi >$. (5 Marks)

ii. Obtain the following commutation relation for a simple Harmonic oscillator (5 Marks)

$$\left[a, \hat{a^+}\right]$$

QUESTION TWO (20 MARKS)

a) Suppose a normalization state ψ is given by $\psi = \sum_{n=0}^{\infty} c_n u_n$ for a simple Harmonic oscillator. Calculate $< \chi >_{\psi(t)}$ and $< p >_{\psi(t)}$after a time t if the system is in a state $\psi(t)$after time $t = 0$. Take

$$x = \sqrt{\frac{h}{2mw}}(a^+ + a)$$

and

$$p = i\sqrt{\frac{mhw}{2}}(a^+ - a)$$

QUESTION THREE (20 MARKS)

A particle of mass m is confined to a medium region $0 \leq x \leq a$ as shown below. At t=0, its normalized wave function is

$$4(x, t = 0) = \sqrt{\frac{8}{5a}}\left(1 + Cos\ \frac{\bar{u}}{a}\right) Sin\ \frac{\pi x}{a}$$

i. What is the wave function of time $t = t_0$ (13 Marks)

ii. What is the average of the system at $t = 0$ and $t = t_0$? (4 Marks)

iii. What is the probability that the particle is found in the left half of the box (i.e. in the region $0 \leq x \leq \frac{a}{2}$ at $t = 0$? (3 Marks)

QUESTION FOUR (20 MARKS)

a) Consider a one-dim bound particle:

i. Show that $\frac{d}{dt}\int_{\infty}^{\infty} \varphi^* (x, t)\varphi(x, t)dx = 0$($\varphi$ need not be a stationary state).

(7 Marks)

b) Show that if the particle is in a stationary state at a given time, then it will always remain in a stationary state (4 Marks)

c) If at $t = 0$, the wave function is constant in the region, $-a < x < a$ and zero elsewhere, express the complete wave function of subsequent time in terms of the Eigen states of the system.

(9 Marks)