MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE (STATISTICS) BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF EDUCATION ARTS

SMS 3357: PROBABILITY AND STATISTICS IV

DATE: APRIL 2024   **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) A fair die is tossed 12 independent times. Determine the probability of the following configuration.

Face	1	2	3	4	5	6
No. of occurrence	1	4	2	0	3	2

(4 Marks)

b) Let $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ be a random vector which is normally distributed, $N_3(\underset{\sim}{\mu}, \Sigma)$ is with mean vector

covariance matrix given below: $\underset{\sim}{\mu} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \Sigma = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{pmatrix}$

find the joint distribution of Z given by $z_1 = y_1 + y_2 + y_3$ and $z_2 = 3y_1 + y_2 - 2y_3$.

(5 Marks)

c) Define the following

i.   Probability generating function   (2 Marks)

ii.   Characteristic function   (2 Marks)

d) Let x be a random variable with generating form $p(s)$ find the generating function of $x + 1$.

(2 Marks)

e) Two random variables x and y have the joint pdf

$$f(x,y) = \begin{cases} kx , & 0 < y < x < 1 \\ 0 , & elsewhere \end{cases}$$

i.   Evaluate the constant k (3 Marks)

ii.   Obtain the marginal distribution of x and y and show that the random variables are not independent.   (4 Marks)

f) Let $x \sim Bin(n, p)$. Using Markor inequality, find and ulper bound $p(x \geq an)$ where $p < a < 1$. Evaluate the bound for $p = {}^1/_2$ and $a = {}^3/_4$.   (4 Marks)

g) A bank teller serves customers standing in a queue one by one suppose that the service time x_i for customer i has mean $E(x_i) = 2$ minutes and $var(x_i) = 1$. Assuming that service time for different bank customers are independent. Let x be the total time the bank teller spends serving so customers. Find $p(90 < x < 110)$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Suppose that x has a Poisson distribution with parameter 7. Obtain the pgf of x and hence find the mean and the variance of x.   (10 Marks)

b) If x is the number scored with the throw of an unbiased die, find the pgf for x.

(4 Marks)

c) Two tetrahedral dice are rolled together once. If x is the number facing down, prove that

$$P(|1x - 7| \geq 3) \leq {}^{35}/_{54}$$ (6 Marks)

QUESTION THREE (20 MARKS)

Given that $\overset{X}{\sim} \sim N(\overset{u}{\sim}, \Sigma)$ with $\mu = \begin{pmatrix} 27 \\ 49 \\ 27 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 36 & 12 & -6 \\ 12 & 16 & 2 \\ -6 & 2 & 4 \end{pmatrix}$

a) Find the distribution of

$$y_1 = x_1 + 2x_3$$
$$y_2 = x_1 - x_2 + x_3$$

(5 Marks)

b) Find the correlation matrix for y (2 Marks)

c) Find the correlation variance of x_1 given x_2 and x_3. (4 Marks)

d) Find the distribution of x_1 given that $x_2 = 40$ and $x_3 = 24$. (5 Marks)

e) Find the condition variance of x_3 when given x_1 and x_2. (4 Marks)

QUESTION FOUR (20 MARKS)

a) Some students did a test in three units and the results were as follows:

Test 1	27	30	27	32	26
Test 2	29	25	40	38	40
Test 3	27	23	34	28	48

Determine the:

    i.   Sample mean vector (2 Marks)

    ii.   Sample variance-covariance matrix (4 Marks)

    iii.   Sample correlation matrix hence interpret (5 Marks)

b) Give a set of independent random variables, show that the characteristic function of their sum is the product of their individual functions. (5 Marks)

c) Let there be a random variable x_p which assumes the value $2^{p-2logp}$ with probability 2^p, $p = 1,2,3 \dots$ examine if the weak law of large numbers hold. (4 Marks)

QUESTION FIVE (20 MARKS)

a) At a particular gas station, gasoline is stocked in a bulk tank each week. Let random variable x denote the proportion of tanks capacity that is stocked in a given week and ley y denote the proportion of the tanks capacity that is sold in the same week. Note that the gas station cannot sell more than what was stocked in a given week implying that the value of y cannot exceed the value of x. possible pdf of x and y is given by

$$f(x,y) \begin{cases} 3x, & 0 \leq y \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

    i.   Obtain the joint cdf of x and y at the point $(x,y) = \left(\frac{1}{2}, \frac{1}{3}\right)$ (5 Marks)

    ii.   Find the probability that the amount of gas sold is less than the amount that is stocked in a given year. (5 Marks)

    iii.   State and prove the central limit theorem. (10 Marks)