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UNIVERSITY EXAMINATIONS 2023/2024

SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS

SMS 3250: PROBABILITY AND STATISTICS III

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Given that X and Y are jointly distributed random variables with pdf.

$$f(x,y)=\begin{cases} bxy & , \quad 1\le x\le 3 \quad and \quad 2\le y\le 4 \\ 0 & , \quad otherwise \end{cases}$$

i. Find the value of b that makes $f(x,y)$ a joint PDF (3 Marks)

ii. Find the marginal density functions of X and Y. (4 Marks)

iii. Determine whether X and Y are stochastically independent (2 Marks)

iv. Find $P\left(X=\frac{x}{y}=2\right)$ (3 Marks)

b) Sample is drawn from a Normal population with mean μ and variance σ^2. Using the moment generating function of the sample $x_1, x_2, \dots, x_n$ find the distribution of the sample mean $\bar{X} = {}^1/_n \sum xi$ (8 Marks)

c) Two discrete random variables X and Y are jointly distributed as:

		X		
		0	1	2
Y	1	0.1	0.1	0.0
	2	0.1	0.1	0.2
	3	0.2	0.1	0.1

i. Determine whether this is a joint PMF (2 Marks)

ii. Find $E[X], E[Y]$ and $E[XY]$ (6 Marks)

iii. Find the covariance of x and y (2 Marks)

QUESTION TWO (20 MARKS)

a) Show that $f(x,y) = \frac{9}{4^{x+y}}, where \begin{matrix} x = 1,2,3,\ldots and \\ y = 1,2,3,\ldots \end{matrix}$

is a joint probability mass function (4 Marks)

b) Ten points are randomly and independently selected from the interval [0,1]. Find the probability that the value closest to 1 exceeds 0.9. (5 Marks)

c) Suppose that the joint pdf of two random variables X and Y is as follows:

$$f(x,y) = \begin{cases} k(x^2+y) & ,0 \le y \le 1-x^2 \\ 0 & , \quad otherwise \end{cases}$$

Determine:

i. The value of constant k (3 Marks)

ii. $P(0 \le x \le 0.5$ (4 Marks)

iii. $P(\times = x/y = 0.5$ (4 Marks)

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, \ldots, X_n$ be n mutually stochastically independent random variables, each with PDF $f_x^{(x)} = 3(1-x^2)$ for $0 < x < 1$. If y is the minimum value of these n variables, find the CDF and PDF of Y. (6 Marks)

b) Suppose that X and Y are Bernouli random variables each with parameter p. Let $Z = x + y$. Find the distribution of z. (4 Marks)

c) Let x and y be jointly distributed with PDF

$$f(x,y)=\begin{cases}6e^{-2x-3y}, & for\ x>0; y>0\\ 0, & otherwise\end{cases}$$

Use the distribution function technique to find the density function of the random variable $z = x + y$. (10 Marks)

QUESTION FOUR (20 MARKS)

a) Suppose that y_1 and y_2 have the joint density.

$$f(y_1, y_2 = \begin{cases}\frac{1}{2\pi\sqrt{0.75}} & \exp\{-{}^{2}/_{3}\,(y_1^2 + y_2^2 - y_1 y_2); \begin{matrix}-\infty < y, < \infty\\ -\infty < y_2 < \infty\end{matrix}\\ 0 & ; otherwise\end{cases}$$

i. Find the marginal density of y, (7 Marks)

ii. Find $P(y_2|y_1 = 1)$. Is this a familiar distribution? (5 Marks)

b) Two discrete random variables X and Y have the following joint PDF.

	X			
Y	1	2	3	4
	0.2	0	0.05	0.15
	0	0.3	0.10	0.2

i. Determine the correlation coefficient between X and Y. comment on your answer. (8 Marks)

QUESTION FIVE (20 MARKS)

a) Given that $f(x,y) = {}^{2}/_{3}\,(x + 2y)$ for $0 \le x \le 1$ and $0 \le y \le 1$

i. Find $E[X/Y]$ (4 Marks)

ii. Find var $(X|Y = 0.4)$ (4 Marks)

b) Let x and Y be a two random variables with joint pdf $f(x,y)$ prove that X and Y ae stochastically independent iff $f(x,y) \equiv g(x)h(y)$ where $g(x)$ and $h(y)$ are positive non-zero functions of X and Y respectively (6 Marks)

c) Given that $X_1, X_2, \dots, X_n$ are n independent random variables with density $N[0,1]$, show that $Y = \sum_{i=1}^{n} x_i^2$ has a chi-square distribution with n degree of freedom. (6 Marks)