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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5139: PARTIAL DIFFERENTIAL EQUATIONS III

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Explain four instances where partial Differential Equation are applied in real life situation (3marks)

b) Which of the following PDEs is linear? Quasilinear? nonlinear? if it is linear, state whether it is homogeneous or not. (5marks)

i. $u_{xx} + u_{yy} - 2u = x^2$

ii. $u_{xy} = u$

iii. $uu_x + xu_y = 0$

iv. $u_x^2 + \log u = 2xy$

v. $u_{xx} - 2u_{xy} + u_{yy} = \cos x$

vi. $u_x(1 + u_y) = u_{xx}$

vii. $(\sin u_x)u_x + u_y - e^x$

viii. $2u_{xx} - 4u_{xy} + 2u_{yy} + 3u = 0$

ix. $u_x + u_x u_y - u_{xy} = 0$

c) Using illustration differentiate between

i. Order of Partial Differential equation (1mark)

ii. Quasilinear Partial Differential equation (2marks)

iii. Well posed and ill posed solution (2marks)

d) Discuss four type of boundary condition in partial differential equations (4marks)

e) Solve the following $\frac{\partial u}{dt} = -3xu$ subject to $u(x,0) = g(x)$ (5marks)

f) Find Green's function for
$$-ku_{xx} = f(x), \quad 0 < x < L$$
$$u'(0) = 0$$
$$u(L) = 0$$

g) Transform the linear second order PDE in two variables is given by (5marks)

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$$

To canonical form

QUESTION TWO (15 MARKS)

a) Consider the following PDE

$$y_t - y_{xx} = 0 \qquad (t,x]\varepsilon(0,T]X[0,1],$$
$$y(0,x) = x \qquad x\varepsilon[0,1]$$
$$y(t,0) = 0 \qquad y(t,1) = 0$$

i. Solve it by method of separation of variable (9marks)

ii. Proof uniqueness of solution (6marks)

QUESTION THREE (20 MARKS)

a) Define Green's formula (3marks)

b) For each of the following problems obtain the function $u(x,t)$ that satisfies the boundary conditions and obtain the PDE.

i. $u_t(x,t) = ku_{xx}(x,t) + x, \quad 0 < x < L$
$$u_x(0,t) = 1$$
$$u(L,t) = t$$

ii. $u_t(x,t) = ku_{xx}(x,t) + x, \quad 0 < x < L$ (3marks)
$$u(0,t) = 1$$
$$u_x(L,t) = 1$$

c) For each of the following problems obtain the function $u(x,t)$ that satisfies the boundary conditions and obtain the PDE. $u_{tt} - c^2 u_{xx} = xt, \qquad 0 < x < L$

d) Subject to each of the boundary conditions

i. $u(0,t) = 1 \; and \; u(L,t) = t$ (3marks)

ii. $u_x(0,t) = t \qquad u_x(L,t) = t^2$ (3marks)

QUESTION FOUR (20 MARKS)

a) Solve the heat equation $u_t = ku_{xx} + x, 0 < x < L$

Subject to the initial condition $u(x,0)x(L-x)$ and each of the boundary conditions

i. $u_x(0,t) = 1 \; and \; u(L,t) = t$ (5marks)

ii. $u(0,t) = 1 \; and \; u_x(L,t) = 1$ (5marks)

b) Consider the following damped wave equation$u_{TT} - c^2 u_{XX} + \beta u_T = \cos \quad \omega t, 0 < x < \pi$

Subject to the initial conditions $\begin{array}{l} u(x,0) = f(x), \\ u_t(x,0) = 0, \end{array}$

And the boundary conditions $u(0,t) = u(\pi,t) = 0$

Solve the problem if β is small $(0 < \beta < 2c)$

QUESTION FIVE (20MARKS)

a) Classify the following second order linear partial differential equations

i. $u_{tt} - c^2 u_{xx} = 0$ (2marks)

ii. $x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} = e^x$ (2marks)

b) Consider the problem $\dfrac{\partial u}{\partial t} + 2u\dfrac{\partial u}{dx} = 0$

$$u(x,0) = f(x) = \begin{cases} 1 \\ 1 \; {}_2 + \frac{x}{L} \end{cases}$$

$$x < 0$$
$$0 < x < L$$
$$L < x$$

i. Determine equations for the characteristics (2marks)

ii. Determine the solution $u(x,t)$ (2marks)

iii. Sketch the characteristics curves (1mark)

iv. Sketch the solution $u(x,t)$ for fixed t (1mark)

c) Solve $\frac{\partial u}{\partial t} + 4u\frac{\partial u}{\partial x} = 0$ subject to $u(x,o) = \begin{cases} 3 & x < 1 \\ 2 & x > 1 \end{cases}$ (5marks)