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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5131: ORDINARY DIFFERENTIAL EQUATION IV

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define the following terms as used in stability of ODEs, and sketch a dynamic system in a plane to represent each case. (6 marks)

i. Stable point
ii. Asymptotically stable point
iii. Unstable point

b) For each of the following ODE's, find all steady states and determine stability of those steady states.

$$\frac{dx}{dt} = x^2 - 2x + 1$$

(3 marks)

$$\frac{dy}{dt} = ry(1-y), r > 0$$

(3marks)

c) Distinguish the four types of equilibrium points and illustrate your answer using diagrams. (8 marks)

d) Determine the stability of the equilibrium point (0, 0) of the system $\dot{x}_1 = -x_1 - x_2$, $\dot{x}_2 = x_1 - x_2^3$ given the lyapunar function $v(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ (3marks)

e) Using Centre Manifold Theorem, determine the stability of the solution of the system at the origin

$$\dot{x} = xy$$
$$\dot{y} = y - x^2$$

(7marks)

QUESTION TWO (15 MARKS)

a) Find the linearization of the system below at each equilibrium point and determine the stability at these point

$$\frac{dx_1}{dt} = -x_1$$
$$\frac{dx_2}{dt} = -x_2 + x_1^2$$
$$\frac{dx_3}{dt} = x_3 + x_1^2$$

(6marks)

b) Define Lyapunov function and identify why it is used to determine stability of Equilibrium points. (5 marks)

c) Define a Hamilton system with n degree of freedom. (4 marks)

QUESTION THREE (15 MARKS)

a) Determine the stability of the system $\dot{x}_1 = -x_2 + \alpha x_1^3$, $\dot{x}_2 = x_1 + \alpha x_2^3$ where α is a real number using a suitable Lyapunov function. (7 marks)

b) Use centre Manifold Theorem to determine the qualitative behaviour near the non hyperbolic critical points at the origin for the system

$$\dot{x} = -x - y - xy$$
$$\dot{y} = 2x + y + 2xy$$

(8 marks)

QUESTION FOUR (15 MARKS)

Compute the one-parameter family of Centre Manifold and describe the dynamics on Center Manifold of the following system. Identify how the dynamic depend on ϵ.

$$\dot{x} = x - 2y + \varepsilon x$$
$$\dot{y} = 3x - y - x^2$$

(15 marks)

QUESTION FIVE (15 MARKS)

a) Define Bifurcation of a fixed point (3 marks)

b) Describe the types of bifurcation using examples and diagrams (12marks)