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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5130: ORDINARY DIFFERENTIAL EQUATION III

DATE: APRIL 2023   **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Show that Lipschitz continuity applies continuity f where f is continuous on the interval $[a, b]$ (5marks)

b) Given the initial value problem

$$y^1(x) = xy$$
$$y(0) = 1$$

Using the picard Heration process, find the third approximate function and the general function $y_n(x)$ (7marks)

c) If the matrix $z(t) = \begin{bmatrix} 0 & 1 \\ -t^{-1} & 1+t^{-1} \end{bmatrix}$, show that $z(t) = \begin{bmatrix} 1+t & e^2 \\ 1 & e^t \end{bmatrix}$ is a Fundamental

matrix of $\overrightarrow{y(t)}$ for $\overrightarrow{y^1}(t) = b(t)\overrightarrow{y}$ and find the solution $\vec{y}(t)$ with initial state $\vec{y}(T) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ at T=2   (10marks)

d) State the Liouville's Theorem   (10marks)

e) Find the exponential matrix of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (4marks)

QUESTION TWO (15 MARKS)

a) Solve the Boundary value problem $\begin{aligned}y(0) &= 1 \\ or \; y^{11} - \lambda^2 y &= 0\end{aligned}$ (7marks)

b) Define Green functions for Boundary value problems (8marks)

QUESTION THREE (15 MARKS)

State and prove Granwell's inequality Theorem

QUESTION FOUR (15 MARKS)

Find the fundamental matrix and the general solution of the system $\dot{x}_1 = 4x_1 + x_2$ given that

initial conditions $\dot{x}_2 = -8x_1 + 8x_2$ $\qquad \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (15marks)

QUESTION FIVE (15 MARKS)

Find the general solution of Nonhomogeneous system of ODES $X^1(t) = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} e^t \\ e^{2t} \end{pmatrix}$

given the initial condition $\vec{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (15marks)