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University Examinations 2023/2024

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER AND BACHELOR OF SCIENCE IN MATHEMATICS

FOURTH YEAR SECOND SEMESTER BACHELOR OF SCIENCE IN EDUCATION

SMA 3352: ORDINARY DIFFERENTIAL EQUATION II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Use the Wronskian to verify that $y_1 = e^x, y_2 = e^{2x}$ *and* $y_3 = e^{3x}$ are linearly independent solutions to $y'''(x) - 6y''(x) + 11y'(x) - 6y(x) = 0$ (3 marks)

b) Solve the Legendre differential equation $(1-x^2)y'' - 2xy' + 6y = 0$ about $x = 0$ (7 marks)

c) Identify and classify the singularity of the equation $3xy'' + 2y' + x^2 y = 0$ (4 marks)

d) Show that (i) $3ydx + (3x + \frac{2}{3}yz)dy + \frac{y^2 dz}{3} = 0$ is an integrable differential equation (6 marks)

e) Use the reduction of order method to solve $x^2 y'' - 3xy' + 4y = 0$ given that x^2 is a solution (6 marks)

i. Solve the equation (4 marks)

QUESTION TWO (20 MARKS)

a) Given that the functions $f(x)\, and\, g(x)$ are infinitely differentiable prove the Leibniz's formula $\frac{d^n}{dx^n}[f(x)g(x)] = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \frac{d^k}{dx^k} f \frac{d^{n-k}}{dx^{n-k}} g$ (6 marks)

b) Consider the equation $2x^2 y'' + xy' - 3y = 0$

i. Classify the singularity (4 marks)

ii. Show that the roots of the corresponding indicial equation are -1 and $3/2$ (10 marks)

QUESTION THREE (20 MARK)

a) Distinguish between an ordinary and a singular point of a second order differential equation (2 marks)

b) Find a power series solution about $x = 0$ $for\ (1 - x^2)y'' - 2xy' + 20y = 0\ given\ y(0) = 0\ and\ y'(0) = 2$ (13 marks)

c) Compute the Legendre polynomial $P_3(x)$ using the Rodrigues formula (5 marks)

QUESTION FOUR (20 MARKS)

a) Show that for a Euler Cauchy equation $ax^2 y'' + bxy' + cy = 0$ we have the solution around $x = 0\ as\ y = x^r$ such that $ar^2 + (b-a)r + c = 0$ hence (6 marks)

ii. Solve the IVP $3x^2 y'' + 2xy' - 4y = 0$ given $y(1) = 0$ and $y^1(1) = 1/3$ (7 marks)

b) Solve the system $\frac{dx}{dt} = 2x - y$, $\frac{dy}{dt} = 3x + 6y$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Solve the equation $x^2 y'' + xy' + (x^2 - 1)y = 0$ about $x = 0$ (20 marks)