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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN DATA SCIENCE

CCD 7153: OPTIMIZATION AND COMPUTATIONAL LINEAR ALGEBRA

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Is $\left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -8 \\ 1 \end{bmatrix} \right\}$. A basis for $\mathbb{R}^3$? (5marks)

b) A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below: (8 marks)

Machine	Time per unit (minutes)			Machine capacity (minutes/day)
	Product 1	Product 2	Product 3	
M_1	2	3	2	440
M_2	4	-	3	470
M_3	2	5	-	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 is Rs. 4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market. Formulate the mathematical (L.P.) model that will maximize the daily profit. (5marks)

c) Explain the following

i. Dominance; (1mark)

ii. Saddle point (2marks)

iii. Infeasible solution (2marks)

d) Compute $p^{-1}AP$ and then $A^n{}_n$ if $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$ and $p = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$ (5marks)

QUESTION TWO (20 MARKS)

a) Find the inverse, if it exists, of the matrix.

$$M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

b) The circle given by the equation $x^2 + y^2 + ax + by + c = 0$ passes through the points $(-2,0), (-1,7)$ and $(5,-1)$. Find a, b, and c using Gaussian elimination (12marks)

QUESTION THREE (20 MARKS)

a) Find values of x if (4marks)

i. $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

ii. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

b) Find the characteristic equation of $\begin{pmatrix} -6 & -14 & -12 \\ 4 & 9 & 6 \\ 1 & 2 & 3 \end{pmatrix}$ (4marks)

c) Find the eigenvalues and Eigenvectors of the matrix below $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (8marks)

d) Find the quadratic for associated with the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (4marks)

QUESTION FOUR (20 MARKS)

a) Consider the following two-person game:

Player 2

		L	R
Player 1	U	1,2	0,1
	D	3,0	X,1

Assume that both players know the value of x, and both know that they know, and so on.

i. For what values of x (if any) is there a Nash equilibrium in which Player 2 chooses R with probability one? Explain, and describe the equilibrium or equilibria in different cases. (3 Marks)

ii. For what values of x (if any) does decision R for Player 2 survive iterated deletion of strictly dominated strategies? Explain. (2 Marks)

b) Two players, Row and Column, are driving toward each other on a one-lane road. Each player chooses simultaneously between going straight (S), swerving left (L), and swerving right (R). If one player goes straight while the other swerves, either right or left, the one who goes straight gets payoff 3 while the other gets —1. If each player swerves to his left, or each swerves to his right, then each gets 0 (remember, they are going in opposite directions). If both go straight, or if one swerves to his left while the other swerves to his right, then the cars crash and each gets payoff — 4.

i. Write the payoff matrix for this game (2 Marks)

ii. Find all of the game's rationalizable strategies for each player. (2marks)

iii. Find all of the game's Nash equilibria in pure strategies. (2marks)

iv. Find a Nash equilibrium in which Row uses a pure strategy and Column mixes between two of his strategies. Clearly identify which strategy or strategies have positive probabilities for each player, and what Column's mixing probabilities are. (Hint: Which of Row's pure strategies could make Column willing to put positive probability on two of Column's pure strategies?) (3 Marks)

v. Find a Nash equilibrium in which both Row and Column mix between two of their strategies. Clearly identify which strategies have positive probabilities for each player, and their mixing probabilities are. (Hint: Pick two pure strategies for each player— because the game is symmetric, it's natural to try the same two strategies for each— and figure out what the mixing probabilities would have to be on just those two strategies. Then compare each player's expected payoff with what he could get by switching to his third strategy).

vi. Find the (unique) Nash equilibrium where each player uses all three of his strategies in a mixture. (Hint: first prove that the probabilities of L and R must be equal in the equilibrium mixture. Then show that for each player the probability of S must be 5/8) (3marks)

QUESTION FIVE (20MARKS)

a) a manufacturer produces three types of plastic fixtures. The time required for molding, trimming and packaging is given in table below. (Times are given in hours per dozen fixtures)

Process	Type A	Type B	Type C	Total time available
Mounding	1	2	$\frac{3}{2}$	12,000
Trimming	$\frac{2}{3}$	$\frac{2}{3}$	1	46,000
Packages	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	2,400
Profit	\$11	\$16	\$15	

b) how many dozen of each type of fixture should be produced to obtain a maximum profit? (10marks)

c) use the simplex method to find the maximum value of $z = 4x_1 + 6x_2$ subject to the constraints

$$-x_1 + x_2 \le 1,$$
$$x_1 + x_2 \le 27, \text{ where } x_1 \ge 0, and\ x_2 \ge 0$$
$$2x_1 + 52 \le 90$$

(10marks)