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University Examinations 2023/2024

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

SMC 3210: NUMERICAL LINEAR ALGEBRA

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Define the following terms (3 marks)

i. A diagonal matrix

ii. A null matrix

iii. Orthogonal matrix

b) Construct a $3x4$ matrix A having $aij = \begin{cases} i+j & when \quad i>j \\ 0 & when \quad i=j \\ i-j & when \quad i<j \end{cases}$ (3 marks)

c) Use the method of substitution to solve the linear system below

$r+2s+t=3$

$2r+3s-t=-6$ (3 marks)

$3r-2s-4t=-2$

d) Solve the given quadratic equation by factorising method (3 marks)

$$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$$

e) If $A = \begin{bmatrix} 4 & 2 & 5 \\ 1 & 3 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$

Evaluate

i. $A - B$ (2 marks)

ii. $2A + 3B$ (2 marks)

f) Find the cofactors of the elements and use them to compute the inverse of the matrix (8 marks)

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & -2 \end{bmatrix}$$

g) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

Find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$ (6 marks)

QUESTION TWO (20 MARKS)

a) Find the inverse of the matrix A using the Gauss- Jordan method (10 marks)

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

b) Prove that a matrix A and its transpose A^T have the same characteristics roots (5 marks)

c) Using Cayley Hamilton theorem find the inverse of the matrix (5 marks)

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

QUESTION THREE (20 MARKS)

a) i. What is a square matrix? (1 mark)

ii. Construct a square matrix P of order $3x3$ in which $p_{ij} = (-1)^{i+j}$ (3 marks)

b) Find the characteristic roots of the matrix $c = \begin{bmatrix} 6 & -4 & 4 \\ -2 & 3 & -1 \\ 4 & -2 & 3 \end{bmatrix}$ (5 marks)

c) Find the eigen values and the corresponding eigen vectors of the matrix (8 marks)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

d) Solve by substitution method the system of linear equations (3 marks)

$$x + 2y + 2x = 3$$
$$2x + 4y + 2x = 8$$
$$x + 2y - x = 10$$

QUESTION FOUR (20 MARKS)

a) Given that $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ find matrix P, such that $P^{-1}AP$ is diagonal matrix

(10 marks)

b) Solve the following system of equations using crammer's rule (10 marks)

$$5x - 7y + z = 11$$
$$6x - 8y - z = 15$$
$$3x + 2y - 6z = 7$$

QUESTION FIVE (20 MARKS)

a) Test the consistency of the equation below

$$2x + 6y = -11$$
$$6x + 20y - 6z = -3 \qquad \text{(5 marks)}$$
$$6y - 18z = -1$$

b) Use matrices to solve (5 marks)

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

c) Show that for each of the following matrices A, the system $Ax = b$ can be solved by Jacobi iteration with guaranteed convergence (5 marks)

$$\begin{pmatrix} 6 & -2 & 4 \\ 2 & -8 & 1 \\ -2 & 0 & 4 \end{pmatrix}$$

d) Find the solution of the following system of equations method by performing the first four iterations with initial conditions (5 marks)

$$18x + y - 2z = 21$$
$$4x + 16y - z = -16$$
$$2x - 3y + 20z = 26$$