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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR FIRST SEMESTER AND FIRST YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5136: NUMERICAL ANALYSIS III

DATE: APRIL 2023 **TIME: 3 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define Weierstrass theorem (2 Marks)

b) What is deflation technique (3 Marks)

c) Obtain the least square approximation for $f(x)=\sqrt{x}$ to $p(x)=a_0+a_1x+a_2x^2$ on [0,1] (5Marks)

d) Obtain the rational approximation of the form $\frac{a_0+a_1x}{1+b_1x}$ *to* e^x (5 Marks)

e) Find a polynomial of degree 3 and use it to approximate $f(x)=\sin x$ near $x_0=0$ and this polynomial to approximate sin 0.1 (5 Marks)

f) Find A^{IO} when $A=\begin{bmatrix}2 & 2\\ 2 & -1\end{bmatrix}$ (5 Marks)

g) Find all the eigenvalues of the matrix $\begin{bmatrix}4 & 3\\ 1 & 2\end{bmatrix}$ using Rutishaurer method (5 Marks)

QUESTION TWO (15 MARKS)

a) We are given the following values of a function of the variable t

t	0.1	0.2	0.3	0.4
f	0.76	0.58	0.44	0.35

Obtain a least squares fit of the form f $=ae^{-3t}+be^{-2t}$ (6 Marks)

b) Find the smallest eigenvalue in magnitude of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using four iterations of the inverse power method (9 Marks)

QUESTION THREE (15 MARKS)

a) Use Chebyshev polynomial to obtain the least squares approximation of second degree for $f(x) = x^4$ on [—1,1] (5 Marks)

b) The table below gives various values of Lagrange polynomial at different points of x. Find the value at x = 1.5 (5 Marks)

c) If $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

i. Find all the eigenvalues and the corresponding eigenvectors (3 Marks)

ii. Verify that $S^{-1}AS$ is a diagonal matrix, where s is the matrix of eigenvectors (2 Marks)

QUESTION FOUR (15 MARKS)

a) Obtain the rational approximation of the form $\dfrac{a_0 + a_1 x}{1 + b_1 x + b_1 x^2}$ to e^x (4 Marks)

b) Determine the second degree interpolating polynomial for $f(x) = \dfrac{1}{x}$ using the point $x_0 = 2, x_1 = 2$ and $x_2 = 4$ (5 Marks)

c) Use the Householder's transformation to reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into tridiagonal matric (6 Marks)

QUESTION FIVE (15 MARKS)

a) Obtain the Taylor series approximation about x = 1, up to second degree terms for the function $f(x) = \dfrac{1}{1+x^2}$. find a bound on the error if this approximation is to be used in [1,14] (4marks)

b) Given the values of $f(x)$ and $f'(x)$

x	$f(x)$	$f'(x)$
-1	1	-5
0	1	1
1	3	7

Estimate the values of $f(-0.5)$and $f'(0.5)$ using the Hermite interpolation. (6 Marks)

c) Consider the matrix $A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 3 \end{bmatrix}$ assuming the dominant eigenvalue is 6 and its associated eigenvector is $v^{(1)} = [1,-1,1]^T$. Use deflation techniques to find the other eigenvalues of A (5marks)