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University Examinations 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER AND BACHELOR OF SCIENCE IN MATHEMATICS

SMA 3456: NUMERICAL ANALYSIS II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Distinguish between interpolation and approximation (2 marks)

b) Let $f(x) = In(1+x), x_0 = 1 and\ x_1 = 1.1$. Apply Lagrange linear interpolation to calculate an approximate value of f(1.04). (4 marks)

c) Obtain the least squares polynomial approximation of degree one for the function $y = \sqrt{x}$ in the interval (0, l) (5 marks)

d) Solve the system of equations by matrix inversion method (5 marks)

$$2x - 3y - 5z = 11$$
$$5x + 2y - 7z = -12$$
$$-4x + 3y + z = 5$$

e) Let $\frac{dy}{dx} = 1 + xy$ with the initial condition that $y = 1, x = 0$. Compute $y(0.1)$ correct to four places of decimal by using Taylor's series method. (7 marks)

f) Find the Eigen values and Eigen vectors of the matrix (7 marks)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

QUESTION TWO (20 MARKS)

a) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = Inx$ defined by the following table of values.

x	$y = Inx$
2	0.69315
2.5	0.91629
3.0	1.09861

Use your polynomial to approximate the value of $In2.7$ and find the absolute error in the approximation. (10 marks)

b) Find the least squares quadratic approximation of the function $f(x) = x^2 + x + 1$ over (-1,1) (10 marks)

QUESTION THREE (20 MARKS)

a) Consider the differential equation $\frac{dy}{dx} = x^3 + y; y(0) = 1,$ compute $y(0.2)$ by Euler's method taking $h = 0.01$ (6 marks)

b) Apply modified Euler's method to determine the value of y when $x = 0.1$ given the differential equation $\frac{dy}{dx} = x^2 + y$ and $y(0) = 1$ with $h = 0.05$ (7 marks)

c) Using Adams-Bashforth predictor-corrector formula, determine $y(0.4)$ given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$ and the data in table below (7 marks)

x	0	0.1	0.2	0.3
y	1	1.0025	1.0101	1.0228

QUESTION FOUR (20 MARKS)

a) Solve the system of equations by Gauss-Jacobi's iteration method (perform 5 iterations) (10 marks)

$$x + 10y + 3 = 6$$
$$10x + y + z = 6$$
$$x + y + 10z = 6$$

b) Find the largest Eigen value and the corresponding Eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} taking\ x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Let $\frac{dy}{dx} = x + y^2$, apply Runge-Kutta 3rd order method to approximate the value of $y\ for\ x = 0.2$ given that y=1 when x=0. Take h=0.1 (10 marks)

b) Determine the Hermite polynomial of 5th degree which fits the following data and hence find an approximate value of $In 2.7$ (10 marks)

x	$y = Inx$	$y' = 1/x$
2.0	0.69315	0.5
2.5	0.91629	0.4000
3.0	1.09861	0.33333