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University Examinations 2023/2024

SECONDYEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE DATA SCIENCE

THIRD YEAR SECOND SEMESTER BACHELOR OF EDUCATION TECHNOLOGY ELECTRICAL AND ELECTRONIC ENGINEERING, BACHELOR OF EDUCATION TECHNOLOGY IN CIVIL ENGINEERING AND BACHELOR OF EDUCATION TECHNOLOGY IN MECHANICAL ENGINEERING

SMA 3303: NUMERICAL ANAYSIS I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Explain two sources of errors in numerical computation (4 marks)

b) If $\pi = \frac{22}{7}$ is approximated as 3.14 . Compute

i. Absolute error

ii. Relative error

iii. Relative percentage error

c) Compute the roots of the function $f(x) = 2x^3 - 3x^2 - 2x + 3$ between the point 1.4 and 1.7 using the bisection method. Take the tolerance to be $\left|x_{j+1} - x_j\right| \leq 10^{-3}$ (5 marks)

d) Compute the zeros of the function $f(x) = 2x^3 - 3x^2 - 2x + 3$ using Newton-Raphson method starting with $x = 1.4$. take the tolerance to be $\left|x_{j+1} - x_j\right| \le 10^{-3}$ (6 marks)

e) Use the trapezoidal rule to evaluate the integral $\int_{0.5}^{3.5} x\sqrt{(16 - x^2)^3}$ assigned increment of $h = 1.0$ (6 marks)

f) Compute the value of $\log_e$ from $\int_0^1 \frac{1}{1+x} dx$ when n=10 using modified Euler formula (5 marks)

QUESTION TWO (20 MARKS)

a) Find an approximate value of the root of the equation $x^3 + x - 1 = 0$ *near* $x = 1$ by method of Regula-False method using the formula twice (5 marks)

b) Find the interpolating polynomial by newton's divided difference formula for the following table and the calculate $f(2.1)$ (5 marks)

x	0	1	2	4
$f(x)$	1	1	2	5

c) Using Lagrange's interpolation formula find a polynomial which passes the points $(0,-12),(1,0),(3,6),(4,12)$ (5 marks)

d) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$ (5 marks)

QUESTION THREE (20 MARK)

a) Given cubic polynomial with the following data points

x	0	1	2	3
$f(x)$	5	6	3	8

Find $\frac{dy}{dx} \& \frac{d^2y}{dx^2} at\ x = 0$

b) Use the Runge-kutta method to approximate $y\ when\ x = 0.1$, given that y=1 when x=0 and $\frac{dy}{dx} = x + y$ (8 marks)

c) The function $y = \sin x$ is tabulated below

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\sin x$	0	0.70711	1.0

Using Lagrang'es interpolation formula, calculate the value of $\sin\frac{\pi}{6}$ (4 marks)

d) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$ (3 marks)

QUESTION FOUR (20 MARKS)

a) In an examination the number of candidates who obtained marks between a certain limit where as follows

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

Find the number of candidates whose scores lie between 45 and 50 (8 marks)

b) Using Lagrange's interpolation formula, find the form of the function y(x) from the table (6 marks)

x	0	1	3	4
y	-12	0	12	24

c) Let $f(x) = x^2 - a.$ show that the newton method leads to the recurrence

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$ (4 marks)

d) Approximate the 1.34579 to three digits by using rounding and chopping (truncation) rules (2 marks)

QUESTION FIVE (20 MARKS)

a) You are working for a company that makes floats for ABC commodes. The floating ball has specific gravity of 0.6g/cm^3 and has a radius of 5.5cm. you are asked to calculate the depth to which the ball is submerged when floating in water. The equation that gives the depth x in meters to which the ball is submerged under water is given by $x^3 - 0.165x^2 + 3.993x10^{-4} = 0. Assume\ f(x) = 0\ and\ x_0 = 0.05m$. Use the Newton-Raphson method to compute the following

i. The depth x to which the ball is submerged under water. Conduct three iteration to estimate the root of the above equation (5 marks)

ii. The absolute relative approximate error at the end of each iteration (4 marks)

iii. The number of significant digit at least at the end of each iteration (3 marks)

b) The following table gives the relation between steam pressure and temperature. Find the pressure at temperature375^0 (8 marks)

Temp	361^0	367^0	378^0	387^0	399^0
Pressure	154.9	167.9	191	212.5	244.2