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University Examinations 2023/2024

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE

SECOND YEAR SECOND SEMESTER BACHELOR OF SCIENCE IN EDUCATION AND BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER AND BACHELOR OF SCIENCE MATHEMATICS

SMA 3212: NUMBER THEORY

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Show that $p|s$ given that $p|s$ and that $q|s$ (4 marks)

b) Using division algorithm, calculate

i. (1911,1001) (3 marks)

ii. [1911,1001] (2 marks)

c) i. Determine the first five integers $mi = p_1.p_2.p_3.....p_i + 1$ where $p_1, p_2, p_3....$ are prime numbers $p_1 = 2, p_2, p_2 = 3, p_3 = 5,....$ in ascending order (5 marks)

ii. show that m_6 and m_7 are not prime numbers (3 marks)

d) i. Define a linear Diophantine equation (2 marks)

ii. State any two possible solutions of the linear Diophantine equation $3x + 2y = 2$

(2 marks)

e) Solve the linear congruence $42X = 50(\text{mod}\,76)$ (4 marks)

f) Define a primitive Pythagorean triple and give two examples (3 marks)

g) State Fermat's last theorem (2 marks)

QUESTION TWO (20 MARKS)

a) Determine the least common multiple (LCM) of 232 and 136 (5 marks)

b) Show that 41 divides $2^{20} - 1$ (5 marks)

c) Solve the Diophantine equation $738x + 621y = 45$ (10 marks)

QUESTION THREE (20 MARKS)

a) i. Define a quadratic residue (modn) (2 marks)

ii. By considering $1^2, 2^2, ...10^2 (\text{mod}11)$ determine the quadratic residues (mod 11) and the non-residue (mod 11) (10 marks)

b) Using the following definition that a belongs to the exponent $k(\text{mod}\,n)$. If k is the smallest positive integer y such that $a^y \equiv 1(\text{mod}\,n)$; show that 5 belongs to the exponent $6(\text{mod}\,7)$. Conclude that the powers of 5 form residues (mod 7) (8 marks)

QUESTION FOUR (20 MARKS)

a) i. Define Euler's phi-function $\phi(n)$ (2 marks)

ii. Determine $\phi(30)$ and list them down (5 marks)

b) i. Prove that if P prime, then $(p-1)! \equiv -1(\text{mod}\,p)$ (7 marks)

ii. Using $p = 13$, verify the above proof (6 marks)

QUESTION FIVE (20 MARKS)

a) i. Define a residue class modulo n (2 marks)

ii. List down residue classes modulo 8 (7 marks)

b) i. Define the Pell's equation (3 marks)

ii. Show that the following rational approximations to $\sqrt{2}$ are all "good" in the sense that $a^2 - 2b^2 = \pm 1$

$\frac{a}{b} = \frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}$ (7 marks)