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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3467: NON-LIFE INSURANCE MATHEMATICS

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Explain why insurance companies make use of run-off triangles. [4 marks]

(b) Describe the key difference between excess of loss and proportional reinsurance. [4 marks]

(c) Explain why claim amounts from general insurance policies are typically modelled using statistical distributions with heavy tails. [4 marks]

(d) The claim amounts on a particular type of insurance policy follow a Pareto distribution with mean 270 and standard deviation 340. Determine the lowest retention amount such that under excess of loss reinsuranceces the probability of a claim involving the reinsurer is 5%. [6 marks]

(e) The run-off triangle below shows the incremental claims incurred on a portfolio of car insurance policies. Calculate the outstanding claims using the basic chain ladder method.

[6 marks]

Development Year

Accident

Year	0	1	2	3
2003	362	272	506	350
2004	444	116	165	
2005	487	195		
2006	518			

(f) Describe how a typical no claims discount system operates in private motor insurance in the UK, indicating the levels of discount typically offered. [6 marks]

QUESTION TWO (20 MARKS)

(i.) Show that:

$$\int_a^b \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(lnx-\mu)^2} dx = e^{\mu+\frac{1}{2}}\sigma^2 \left[\Phi\left(\frac{lnb-\mu-\sigma^2}{\sigma}\right) - \Phi\left(\frac{lna-\mu-\sigma^2}{\sigma}\right)\right]$$ (8marks)

A general insurance company writes claims, whose amounts have a lognormal distribution, with mean 300 and standard deviation 400. The insurance company purchases excess of loss reinsurance with retention 500 per claim.

(ii.) Calculate the average expected claim size payable by the insurance company. [8 marks]

Next year, claim inflation is 10%, but the retention amount remains the same.

(iii.) Explain whether the average expected claim size payable by the insurance company next year would increase by 10%. [4 marks]

QUESTION THREE (20 MARKS)

A European country has just introduced a no claim discount system with three levels:

Level 1: 0% discount

Level 2: 25% discount

Level 3: 50% discount

The rules for movements between the levels are as follows:

1. Following a claim free year, the discount increases by one level (or remains at level 3).
2. Following a year in which exactly one claim was made, the discount decreases by one level (or remains at level 1).
3. Following a year in which more than one claim is made, the discount returns to level 1. The insurer classifies motorists into three categories, which are considered to have the following expected annual claim frequencies λ :

   High risk: 0.4

   Medium risk: 0.2

   Low risk: 0.1

The numbers of claims made in each year under each policy can be assumed to have a Poisson distribution.

(a) Write down the transition matrix (in terms of the claim frequency λ) showing the probabilities pij that a policyholder on discount level j in one year will be on discount level j the following year. [2 marks]

(b) Derive an algebraic expression for the average discount level for a group of policies with the same risk level when the group has reached a stable state. [12marks]

(c) Hence calculate the long-term average discount for policyholders in each of the three risk categories. Comment on your answers. [6 marks]

QUESTION FOUR (20 MARKS)

The random variable X represents the amount of a single claim arising from an insurer's portfolio. Under an excess of loss reinsurance treaty, a reinsurer pays the excess of each claim amount over £50,000, so that, for each claim, the reinsurer pays:

$$\begin{cases} 0 \; if \; X \leq 50{,}000 \\ X - 50{,}000 \;\; if \; > 50{,}000 \end{cases}$$

The amounts paid by the reinsurer, ie x — 50, 000, for the five most recent claims that exceed ed £50,000 were £153,000, £376,000, £120,000, £20,000 and £108,000

Assume the density function of X is given by:

$$f(x;\theta) = \text{e x } 2^{\theta} \text{ x } 10^{5\theta} (2 \text{ x } 10^5 + \text{x})^{-\theta-1}, X > 0$$

where e is an unknown parameter.

(i) Find θ, the maximum likelihood estimate of e. [7 marks]

(ii) Estimate the standard error of $\hat{\theta}$. [5 marks]

(iii) Use $\hat{\theta}$ to estimate the average amount payable by the reinsurer on claims that exceed £50,000. [2 marks]

(iv) If all claim amounts increase by 10% as a result of inflation, but the retention limit remains at £50,000, estimate the average amount payable by the reinsurer on claims that exceed £50,000. [6 marks]

QUESTION FIVE (20 MARKS)

(i.) Define the term 'loss ratio' as used in the Bornhuetter—Ferguson method for estimating outstanding claim amounts. [2 marks]

The run-off triangle below shows cumulative claims incurred on a portfolio of insurance policies.

	Development		
Accident Year	0	1	2
2018	798	915	1,320
2019	820	1,412	
2020	1,016		

Annual premiums written for accident year 2019 were 1,520 and the ultimate loss ratio is assumed to be 92.5%. Claims can be assumed to be fully run off by the end of development year 2.

(ii.) Calculate the total claims arising from accidents in 2019, using the Bornhuetter—Ferguson method. [8 marks]

1 year later, an unexpected event has resulted in higher claims than expected. The run-off triangle is now as shown below.

Development

Accident Year	1	2	
2018	798	915	1,320
2019	820	1	
2020	1,016		

(iii.) Calculate the revised total claims arising from accidents in 2019, using the Bornhuetter— Ferguson method. [6 marks]

(iv.) Discuss the implications of your answer to part (iii) for the insurance company. [4 marks]