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UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS, BACHELOR OF SCIENCE (MATHEMATICS AND PHYSICS) AND BACHELOR OF SCIENCE (PHYSICS)

SPH 3352: MATHEMATICAL PHYSICS III

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define a group. (4 Marks)

b) Show that $(\mathbb{Z}, +)$ is a group. (4 Marks)

c) Define a cyclic group. (2 Marks)

d) Define a permutation group. (2 Marks)

e) Define the term Manifold (2 Marks)

f) Give examples of manifolds including their respective dimensions (4 Marks)

g) What is a tensor? (2 Marks)

h) What is the rank of a tensor? (2 Marks)

i) Scalars and vectors are tensors of what ranks? (2 Marks)

j) The tensor A_{ij}^{k} is of what rank? (1 Mark)

k) Explain what is meant by a tensor A_{ij} being symmetric if. (1 Mark)

l) Explain what is meant by a tensor A_{ij} being anti-symmetric if. (1 Mark)

m) Which of the following tensors are contravariant, covariant and mixed: A^{ij}, B_{ij}, C_k^{ij}

(3 Marks)

QUESTION TWO (20 MARKS)

a) Give the relation for the Levi-Civita connection formula (2 Marks)

b) Calculate the Levi-Civita connection of Euclidean metric of a plane in Cartesian coordinates:

i. Cartesian coordinates (3 Marks)

ii. Polar coordinates: $G = dr^2 + r^2 d\phi^2$ (5 Marks)

iii. $G = R^2(d\theta^2 + sin\theta^2 d\phi^2)$ on the sphere (5 Marks)

iv. Using the geodesic equation, show that geodesics on a plane are straight lines

(5 Marks)

QUESTION THREE (20 MARKS)

a) Consider the function $f = xy$ and differential forms $\sigma = xdy + ydx \; and \; \omega = zdx + xdy$. Calculate the differential forms $df(\omega); \sigma\wedge\omega$ and $d(\sigma\wedge\omega)$. (8 Marks)

b) Consider in $\mathbb{R}^2$a triangle ΔABC with vertices at the points $A = (5,1), B = (1,6), C\ (5,1)$ and differential one-form $\omega = xdy - ydx$. By using Stokes' theorem calculate the integral of $1-$form ω over the boundary of the ΔABC (7 Marks)

c) Convert the one-form

$$\omega = (x^2 + y^2)dx + zdy + dz$$

to cylindrical coordinates (5 Marks)

QUESTION FOUR (20 MARKS)

a) Define the general linear group GL_n (2 Marks): Given a vector space V, the general linear group, often denote as GL_n , represents the set of all invertible $n \times n$ matrices over a given field.

b) Define a representation of a group G (2 Marks)

c) i. The rotation matrix is expressed as,

$$R_k = \begin{pmatrix} cos\left(\frac{2\pi}{n}\right) & -sin\left(\frac{2\pi k}{n}\right) \\ \sin\left(\frac{2\pi k}{n}\right) & \cos\left(\frac{2\pi}{n}\right) \end{pmatrix}$$

Where k is an integer that represents the number of times you perform a transformation, and n is the total number of distinct orientations that are symmetrically equivalent. Considering the symmetry transformations on an equilateral triangle, write down the expressions for R_1, R_2, R_3. (3 Marks)

ii. Which of the representations R_1, R_2, R_3 represents the identity element of the group? Explain your answer. (3 Marks)

iii. Show that $R_1 R_2 = R_3$ (5 Marks)

d) Considering the permutation of three elements (1, 2, 3). The elements 1, 2,3 are represented as (100), (010) and (001) respectively. Using this representation, obtain the matrices for the identity permutation, Permutation (13), Permutation (12) and Permutation (123).

(5 Marks)