



MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE IN PHYSICS, BACHELOR OF SCIENCE (MATHEMATICS
AND PHYSICS) AND BACHELOR OF SCIENCE (PHYSICS)

SPH 3254: MATHEMATICAL PHYSICS II

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Define the modulus of a complex number $z = x + iy$ (1 Mark)
- b) i. Show that $e^{i\pi} = -1$ (3 Marks)
ii. Find the $u(x, y)$ and $v(x, y)$ of f for $f(z) = z^2$ where $z = x + iy$. (3 Marks)
- c) Differentiate between a Fourier series and a Fourier Transform. (2 Marks)
- d) Define the following terms.
- i. Zero of a function (1 Mark)
 - ii. Singular Point (1 Mark)
 - iii. Degree of a differential equation (1 Mark)
 - iv. Order of a differential equation (1 Mark)
 - v. Homogeneous differential equation (1 Mark)
 - vi. Non-Homogeneous Differential Equation (1 Mark)
- e) Determine the order and degree of the differential equation (2 Marks)
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$$\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos(3x)$$

f) Identify which of the following differential equations are homogeneous or non-homogeneous

(3 Marks)

$$\frac{xdy}{dx} + y = 0, x^4 + \frac{y^4dy}{dx} = 0, \frac{xydy}{dx} + 2x + y^2 = 0$$

g) Differentiate between a General solution and a particular solution of a differential equation.

(2 Marks)

h) Find the general solution of the equation $\frac{dy}{dx} = x^2y + y$

(4 Marks)

i) Find the partial differential equation of $u = f(x^2 - y^2)$

(4 Marks)

QUESTION TWO (20 MARKS)

a) Show that the function $f(z) = e^z$ is differentiable at any point given that $z = x + iy$

(5 Marks)

b) Evaluate $\int_C \frac{z^2-4z+4}{z+i} dz$ using the Cauchy-Integral Formula where $|z|$

(2.4 Marks)

c) Evaluate $\int_C \frac{z^2+5}{z-3} dz$ where $|z| = 4$ using the Cauchy Integral Formula.

(4 Marks)

d) Find the residue of, $f(z) = \frac{e^z}{(z-2)^4}$

(3 Marks)

e) Compute $\int_C \frac{5z-2}{z(z-1)} dz$ using residues for C a circle $|z| = 2$

(2.4 Marks)

QUESTION THREE (20 MARKS)

a) Any reasonably well-behaved periodic function can be written as a sum of sines and cosines, or rather as a Fourier Series:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt)$$

Multiplying the above equation by either $\sin mt$, $\cos mt$ or 1 and integrating in the range $[0, 2\pi]$ show that



i. $b_m = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(mt) dt$ (4 Marks)

ii. $a_m = \frac{1}{\pi} \int_0^{2\pi} f(t) \cos(mt) dt$ (4 Marks)

iii. $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(t) dt$ (4 Marks)

b) Find the Fourier series of the function (8 Marks)

$$f(\theta) = \begin{cases} -1 & -\pi < \theta < 0 \\ 1 & 0 < \theta < \pi \end{cases}$$

QUESTION FOUR (20 MARKS)

a) Given a differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Write down the expression for the integrating factor(IF) for this equation. (2 Marks)

b) 20 grams of salt are dissolved into a beaker containing 1 litre of a certain chemical. The mass of salt, M grams, which remains undissolved t seconds later, is modelled by the differential equation $\frac{dM}{dt} + \frac{2M}{20-t} + 1 = 0, t \geq 0$

Show clearly that $M = \frac{1}{10}(10 - t)(20 - t)$ (8 Marks)

c) Find the nature of the following PDEs (9 Marks)

i. $4u_{xx} + 4u_{yy} + 2u_x - u_y = 0$

ii. $u_{xx} - y^4 u_{yy} = 2y^3 u_y$

iii. $x^2 u_{xx} + 2xy u_{xy} + (1 + y^2) u_{yy} - 2u_x = 0$

iv. Give examples of an Elliptic and hyperbolic partial differential equation (1 Mark)

