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University Examinations 2023/2024

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER BACHELOR OF SCIENCE MATHS(PHYSICS)

SMA 3254: LINEAR ALGEBRA II

DATE: APRIL 2024 **TIME:2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Show that $T:R^3 \to R^2$ defined by $T(x,y,z)=(x+y+z)$ is a linear transformation (4 marks)

b) Let $g=\{g_1,g_2,g_3\}$ be a basis for $\mathbb{R}^3$ and $\{g_1=(1,1-1), g_2=(0,1,1) g_3=(0,0,1)\}$. Find the coordinate vector of $V=(4,2,-1)$ relative to the basis g (5 marks)

c) Determine whether the following two matrices are similar

$A=\begin{bmatrix}-2 & 4\\ 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2\\ -2 & -2\end{bmatrix}$ (4 marks)

d) Let $A=\begin{bmatrix}4 & 1\\ 2 & 3\end{bmatrix}$, $f(t)=3t^2-12t+1$. Find $f(A)$ (5 marks)

e) Let T be linear mapping on $\mathbb{R}^2$ defined by $T(x, y) = (2x - 3y, x + 2y)$ Find the matrix of T in the basis $\{e_1 = (1,0), e_2 = (0,1)\}$ (5 marks)

f) Compute the determinant of $A = \begin{bmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{bmatrix}$ (7 marks)

QUESTION TWO (20 MARKS)

a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear mapping defined by $T(1,1,1) = (1,0) = (2,-1)$ *and* $T(1,0,0) = (4,3)$.*Find* $T(x, y, z)$ the compute $T(2,-3,5)$

(8 marks)

b) Find the characteristics polynomial, minimal polynomial and the eigenvalues of the matrix

$A = \begin{bmatrix} 5 & 2 & 2 \\ -2 & 1 & -2 \\ 2 & 2 & 5 \end{bmatrix}$ (12 marks)

QUESTION THREE (20 MARKS)

a) Consider the basis $\{f_1 = (1,2), f_2 = (2,3)$ *and* $g_1 = (1,3), g_2 = (2,5)\}$ two basis of $\mathbb{R}^2$

i. Find the transition matrix P from $\{f_1\}$ *to* $\{g_1\}$ and the transition matrix Q from $\{g_i\}$ *to* $\{f_i\}$ (12 marks)

ii. Verify that $Q = P^{-1}$ (1 mark)

iii. Show that $P^{-1}[V]_f = [V]_g$ (2 marks)

b) The set $\langle 1, t, 3t, \cos 3t \rangle$ is a basis a a vector space V *of* $v \to v$. Let D be the differential operator on V, that is $D(f) = \frac{df}{dt}$. Find the matrix of D in the given basis (5 marks)

QUESTION FOUR (20 MARKS)

a) Compute the determinant of $\begin{pmatrix} 4 & 0 & -7 & 3 & 5 \\ 0 & 0 & -2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 3 \end{pmatrix}$ (5 marks)

b) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$

i. Find the matrix representation of T relative to the following basis of $R^3 \, and \, \mathbb{R}^2$

$(f_1 = (1,1,1), f_1 = (1,1,0), f_1 = (1,0,0)) \, and \, (e_1 = (1,3), e_2 = (1,4))$ (10 marks)

ii. Verify that for any vector $v \epsilon \mathbb{R}^3 [T]_f^e [v]_f = [T(v)]_e$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Diagnonalize $B = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix}$ (12 marks)

b) Show that $T : P_2 \to P_2$ defined by $T(a_0 + a_1 x + a_2 x^2 = (a_0 + 1) + a_1 x + a_2 x^2$ is not a linear transformation (4 marks)

c) Given the matrix $B = \begin{pmatrix} 4 & -2 & 3 \\ 2 & -1 & 0 \\ 1 & -3 & -5 \end{pmatrix}$ compute the minors and cofactors of the elements in the first column (4 marks)