UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

SMS 3460: FINANCIAL TIME SERIES AND RISK MANAGEMENT

DATE: APRIL 2023   **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Briefly explain five characteristics of financial time series   [5 marks]

(b) State three key properties of Value at Risk (VaR) in financial risk management.   [6 marks]

(c) Describe three ways that a GARCH model can generate a heavy tailed distribution.   [6 marks]

(d) Given ARCH (1) model $a_t = \sigma_t e_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$ where $e_t \sim N(0,1)$ and $a_0 > 0, \alpha_1 \geq 1$ GARCH (11) model. Show that $\text{var}[a_t] = \dfrac{\alpha_0}{1-\alpha_1}$ [6 marks]

(e) State and describe the GARCH-M model   [4 marks]

(f) State the square root of time rule in value at Risk (VaR) calculation under Risk metrics   [3 marks]

QUESTION TWO (20 MARKS)

Given $a_t = \sigma_t e_t, \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta \sigma_{t-1}^2$ where $e_t \sim N(0,1)$ is a GARCH (Il) model. Show that

i. $E[a_t] = 0$ [3 marks]

ii. $Var[a_t] = \dfrac{\alpha_0}{1 - \alpha_1 - \beta_1}$ [8 marks]

iii. The process in leptokurtic [9 marks]

QUESTION THREE (20 MARKS)

a) Consider and ARCH (i) process X_t given as; $X_t \sqrt{\beta_0 + \beta_1 X_{t-1}^2 + e_t}$

where $e_t \sim IID\ N(0, \sigma^2)$ and independent of x_{t-1}

i. Find the stationary variance of X_t, giving necessary conditions on β_0 and β_1

[8 marks]

ii. Show that X_t, is non-normal even if e_t is assumed to be normal. [6marks]

iii. Show that $X_t - \beta_1 X_{t-1}^2$ is a white noise with mean zero [6marks]

QUESTION FOUR(20MARKS)

a) Give three advantages of EGARCH models over the GARCH models. [6 marks]

b) Consider a financial time series model

$X_t = \sigma_t e_t$.

where $e_t \sim IID\ N(0, \sigma^2)$ and X_t is a stationary process.

(i.) Define a GARCH) (p, q) process [6 marks]

(ii.) Show that X_t^2 follows an ARMA (p, q) process with non-zero mean [8 marks]

QUESTION FIVE (20 MARKS)

a) Explain the stages in Financial Risk Management [8 marks]

b) Imagine you are a financial risk manager with two stocks namely:

Stock X: Initial investment: $100,000 Expected return: 8% Standard deviation: 12%

Stock Y: Initial investment: $150,000 Expected return: 12% Standard deviation: 18%

Assume a correlation coefficient (p) between the two stocks of 0.5. We want to calculate the I-day Var at a confidence level of 95%   (12marks)