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UNIVERSITY EXAMINATIONS 2023/2024

SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3263: FINANCIAL MATHEMATICS II

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

- a) Define the following terms:
- i. Arbitrage opportunity [2 Marks]
 - ii. Call option. [2 Marks]
- b) Describe the investment characteristics of convertible stocks. [4 Marks]
- c) An asset has a current price of £1.20. Given a risk-free rate of interest of 5% pa effective and assuming no arbitrage, calculate the forward price to be paid in 91 days. [4 Marks]
- d) A 91 -day government bill is discounted at a simple rate of discount of 10% pa. Calculate the annual effective rate of interest earned on this investment. [4 Marks]
- e) Explain the main differences between deterministic and stochastic interest rates models. [4 Marks]
- f) An equity is expected to pay its first dividend in exactly 2 years' time. It is assumed that this dividend will be \$0.20 per share. Subsequent annual dividends are assumed to grow at 6% p.a. compound for the following 10 years, and at 3% p.a. compound in perpetuity thereafter. Calculate, showing all working, the price of the share to the nearest \$0.01, that would give an effective rate of return of 7% p.a. [5 Marks]



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- g) A bond is issued at time $t = 0$ at a price of \$107.60 per \$100 nominal. The bond pays coupons of 6% p.a., annually in arrears, and will be redeemed at par in 3 years' time. The 2-year par yield at time $t = 0$ is 6.5% p.a. The 1-year forward rate of interest at time $t = 1$ year is 4.5% p.a. effective. Calculate, showing all working and assuming no arbitrage, the implied 1-year, 2-year and 3-year annual effective spot rates. [5 Marks]

QUESTION TWO (20 MARKS)

- a) Define the following terms:
- spot rate payments [2 Marks]
 - Forward rate [2 Marks]
 - Yield to maturity [2 Marks]
- b) A fund earns an annual rate of return i_t , with the rate of return in any year being independent of the rate in any other year. The distribution of $\log(1 + i_t)$ is normal with parameters μ and σ^2 . The mean of it is 5% and the standard deviation is 3%.
- Calculate μ and σ^2 [4 Marks]
 - Calculate the probability that the fund return for any year is between 1% and 3%. [4 Marks]
 - Comment on your answer to part (ii). [2 Marks]
A sum of £10,000 is invested into the fund.
 - Calculate the probability that the accumulated value of the fund at the end of 3 years is less than £11,000. [4 Marks]

QUESTION THREE (20 MARKS)

- a) An insurance company has a continuous payment stream of liabilities to meet over the coming 20 years. The payment stream will be at a rate of £10 million per annum throughout the period. Calculate the duration of the continuous payment stream at a rate of interest of 4% per annum effective. [6 Marks]
- b) The following information about the term structure of the interest rates is given:



Term (in years)	Spot rate (% p.a.)
1	2.3
2	2.8
3	X
4	3.5
5	4.5

A 1 -year zero-coupon bond will be issued at time 3 and has a theoretical price of \$95 per \$100.

i. Calculate, showing all working and assuming no arbitrage:

a) X, the 3-year spot rate. [3 Marks]

b) the 2-year discrete forward rate starting at time 2. [3 Marks]

A 5-year bond is issued paying annual coupons at 9% p.a. and is redeemable at 110% of the par value.

ii. Show that the annual gross effective redemption yield on this bond is approximately 4.29%.

[4 Marks]

iii. Comment on the numerical value of the annual gross effective redemption yield in part

[4 Marks]

QUESTION FOUR (20 MARKS)

A government issues a fixed interest bond paying coupons at a rate of 9% per annum, payable half-yearly in arrears. The bond is to be redeemed at \$110 per \$100 nominal on any coupon payment date from 10 to 15 years after issue. The date of redemption is at the discretion of the government. Investor A is subject to income tax at 25% and capital gains tax at 30%, and wishes to achieve a net redemption yield of at least 6% per annum effective.

i. Calculate, showing all working, the maximum price per \$100 nominal that Investor A should offer for this bond on issue. [7 Marks]

Investor A purchases the bond at the price determined in part (i). Three years after issue, immediately after a coupon payment has been made, Investor A decides to sell the bond to Investor B. Investor B is subject to income tax at 10% and capital gains tax at 35%, and wishes to obtain a net redemption yield of at least 8% per annum effective.



- ii. Calculate, showing all working, the maximum price per \$100 nominal that Investor A can expect to receive from Investor B. [7 Marks]

The bond is sold to Investor B at the price determined in part (ii).

- iii. Calculate, using linear interpolation, the net effective annual redemption yield that will be obtained by Investor A. You must show all your working. [6 Marks]

QUESTION FIVE (20 MARKS)

- i. a) Explain what is meant by the "expectations theory" explanation for the shape of the yield curve. [4 Marks]
- b) A company has to meet liability payments at the end of each of the next 25 years. The payment at the end of the first year is £1.2 million. Thereafter, the payments are expected to increase at a fixed rate of 4.3902% p.a. compound.
- ii. Calculate, showing all working, the duration of the company's liabilities using an effective rate of interest of 7% p.a. [10 Marks]
- The rate at which payments are expected to increase has been revised, and is now greater than 4.3902% p.a.
- iii. Identify, with reasons, whether your answer to part (i) will increase, decrease or remain unchanged. Do not perform further calculations. [6 Marks]

