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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3163: FINANCIAL MATHEMATICS I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) For each of the following calculate the equivalent effective annual rate of interest:

i. An effective rate of interest of 12.7% paid every 2 years [2 Marks]

ii. An effective rate of discount of 5.75% pa [2 Marks]

iii. A force of interest of ½% per month [2 Marks]

b) A general insurance company has set up a generalised cashflow model for the claims payments it will pay arising from its portfolio of car insurance policies. Describe the cashflows with reference to the certainty and uncertainty of size and timings of payments from the insurance company's point of view. [5 Marks]

c) An investor receives payments half-yearly in arrears for 20 years. The first payment is £250, and each payment is 2% higher than the previous one. The interest rate is 6% pa effective for the first 10 years and 4% pa effective for the final 10 years. Calculate, showing all workings, the present value of the payments. [5 Marks]

d) In return for a fixed initial deposit, an investor receives a continuously payable annuity for a term of 15 years. The annual rate of payment is 50 in the first year, and the rate of payment increases in each subsequent year. The investor can select either:

Option 1: the rate of payment increases by 2 at the end of each year,

Option 2: the rate of payment increases by 3% pa compound at the end of each year. Determine which option would provide the better deal for the investor at an annual effective interest rate of 7%. [6 Marks]

e) Explain when you would use real and money rates of interest. Give an example of when each rate of interest would be used. [4 Marks]

f) Calculate the length of time it will take £800 to accumulate to £1,000 at a simple rate of interest of 4% pa. [4 Marks]

QUESTION TWO (20 MARKS)

The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 + 0.005t & 0 \leq t < 6 \\ 0.045 - 0.0025t & 6 \leq t < 8 \\ 0.04 & 8 \leq t \end{cases}$$

a) Derive expressions in terms of t for the accumulated amount at time t of an investment of 1 at time 0. [8 Marks]

b) i. Calculate the value at time 0 of £100 due at time 9. [3 Marks]

ii. Calculate the annual effective rate of discount implied by the transaction in (a). [3 Marks]

c) A continuous payment stream is received at a rate of $45e^{0 \bullet 01t}$ units per annum between time 10 and time 15. Calculate, showing all workings, the present value at time 4 of this payment stream. [6 Marks]

QUESTION THREE (20 MARKS)

a) i. Prove that $(Ia)_{n\rceil} = \frac{\ddot{a}_{n\rceil} - nv^n}{i}$ [6 Marks]

ii. An annuity payable monthly in arrears has a first payment of £300, with subsequent payments decreasing by £10 each month, until a final payment of £70 is made in two years' time. Calculate the present value of the payments from this annuity using an effective rate of interest of 6% pa. [6 Marks]

b) Discuss the features of Eurobonds [8 Marks]

QUESTION FOUR (20 MARKS)

a) Discuss the features of Zero Coupon Bonds [6 Marks]

b) Assuming a rate of interest of 6% pa, calculate the present value as at 1 January 2008 of the following annuities, each with a term of 25 years:

c) i. an annuity payable annually in advance from 1 January 2009, initially of £3,000 pa, and increasing by £500 pa on each subsequent 1 January [4 Marks]

ii. an annuity as in (i), but only 10 increases are to be made, the annuity then remaining level for the remainder of the term. [4 Marks]

d) An investor is to receive a special annual annuity for a term of 10 years in which payments are increased by 5% compound each year to allow for inflation. The first payment is to be £1 ,000 on 1 November 2009. Calculate the accumulated value of the annuity payments as at 31

a. October 2026 if the investor achieves an effective rate of return of 4% per half year. [6 Marks]

QUESTION FIVE (20 MARKS)

The force of interest is a function of time and at any time t (measured in years) is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \leq t < 8 \\ 0.07 & 8 \leq t \end{cases}$$

i. Derive, and simplify as far as possible, expressions for $v\ (t)$, where $v\ (t)$ is the present value of a unit sum of money due at time t. [8 Marks]

ii. You should consider separately the cases $0 \leq t < 8$ and $8 \leq t$.

a) Demonstrate that it will take more than 8 years for an investment to double in value. [3 Marks]

b) Calculate, showing all working, the exact time in years it will take for an investment to double in value. [3 Marks]

The inflation rate over the first 10 years has been constant at 2% p.a. convertible quarterly.

iii. Calculate, showing all working, the real rate of return obtained on an investment made over this period. Express your answer as a percentage nominal rate p.a., convertible quarterly. [6 Marks)