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University Examinations 2023/2024

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN DATA SCIENCE

SMC 3212: DISCRETE STRUCTURES

**DATE: APRIL 2024**   **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Distinguish between permutations and combinations   (2 marks)

b) Represent the set $A = \{x : x \in N \text{ and } 1 < x < 7\}$ in list form where N represents natural numbers   (2 marks)

c) Find the dual of the Boolean equation $X + (Y + Z) = (X + Y) + Z$ (2 marks)

d) If $f(x) = \sqrt{2x + 5}$ *and* $g(x) = x^2 + 5$, calculate *fog* and *gof*   (4 marks)

e) In how many ways can 10 committee members sit around a circular conference table if the chairman must sit between the secretary and the treasurer   (4 marks)

f) Find the adjacency matrix A of the graph below   (5 marks)
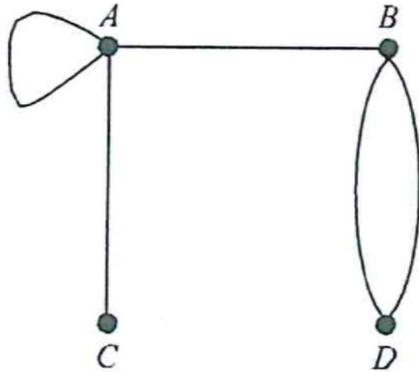
g) Prove by mathematical induction that for any positive integer $n, 6^n - 1$ is divisible by 5

(5 marks)

h) Find the output of the Boolean function represented by the logic circuit   (6 marks)
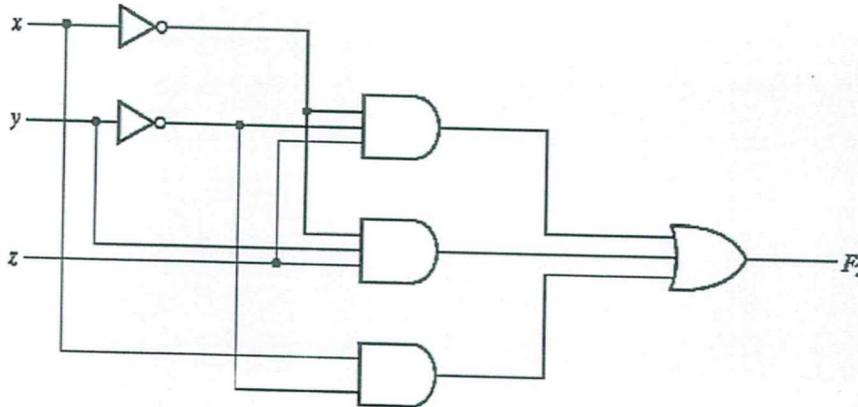
QUESTION TWO (20 MARKS)

a) A survey of 120 randomly selected people, it was found that;

   65 read Newsweek magazine

   45 read time magazine

   42 read fortune magazine

   20 read both newsweek and time

   25 read both newsweek and fortune

   15 read both time and fortune

8 read all the three magazines

Let N,T and F denote the set of people who read Newsweek, Time and Fortune respectively

i.   Draw a Venn diagram to represent this information   (5 marks)

ii.   Find the number of people who read exactly one magazine   (2 marks)

iii.   Find the number of people who read at least one of the three magazines (2 marks)

b) Which of the following sets are finite and which ones are infinite?   (4 marks)

    i.   Set of lines which are parallel to a given line

    ii.   Set of animals on the earth

    iii.   Set of natural numbers less than or equal to fifty

    iv.   Set of points on a circle

c) Let $A = [\{1,2,3\},\{4,5\},\{6,7,8\}]$. Find $n(A)$ i.e cardinality of set A   (3 marks)

d) Let $\{2,5,7\}$, find $P(B)$, the power set of B   (4 marks)

QUESTION THREE (20 MARKS)

a) Distinguish between an isomorphic and homoeomorphic graphs   (2 marks)

b) Consider a simple connected graph G with n vertices and n edges $(n > 2)$. Then classify

the following statements as TRUE or FALSE   (4 marks)

    i.   G has no cycle

    ii.   G has at least one cycle

    iii.   The graph obtained by removing any edge of G is not connected

    iv.   The graph obtained by removing any edge of G is connected

c)   Draw a multigraph G corresponding to the adjacency matrix   (6 marks)

$$A = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

d) Find a minimal spanning tree T for the weighted graph below   (8 marks)
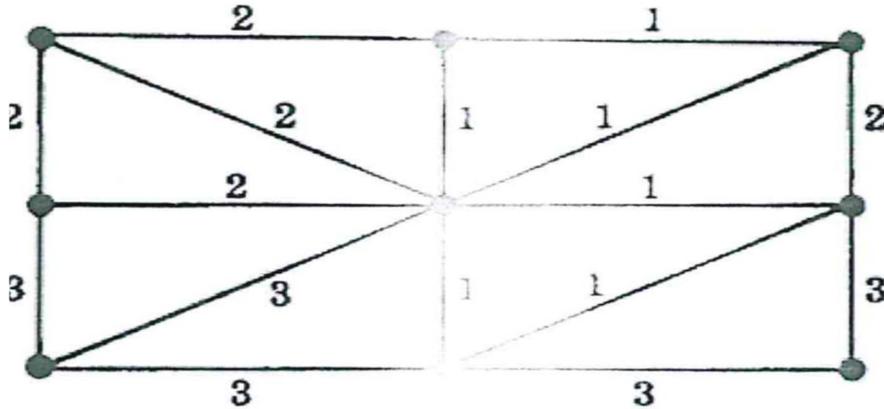
QUESTION FOUR (20 MARKS)

a) Distinguish between a theorem and a lemma   (2 marks)

b) Let A=1,B=0,C=1

   Draw a logic gate circuit for the output state Q=A.B+C   (4 marks)

c) Determine the number of permutations of the letters of the word

   RECURRENCERELATION   (4 marks)

d) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. Ow many ways are there to select a committee to develop a discrete mathematics course if the committee is to consist of three faculty members from the mathematics department and four from the computer science department

                                                                                (4 marks)

e) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?   (6 marks)

QUESTION FIVE (20 MARKS)

a) Use a truth table to prove that $A(A'+B) = AB$ (5 marks)

b) Prove by mathematical induction that $1^3 + 2^3 + ...n^3 = \dfrac{1}{4}n^2(n+1)^2$ (5 marks)

c) The first four terms of the binomial expansion of $(1+2x)^8$ in ascending powers of x are

$1+ax+bx^2+cx^3$ find the values of integers a, b and c (5 marks)

d) A particular college has a list of approved prerequisite course

MATH	SCIENCE	ENGLISH	OTHER
Math 301	Biology 301	English 301	Option A
Math 302	Chemistry 301		Option B
	Physics 301		Option C
			Option D
			Option E

Five courses are required for admission to the college. Math 301 (or math 302) and English 301 are mandatory requirements, and at least one science must be selected as well. How many different ways could a student select five courses?   (5 marks)