



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF  
SCIENCE IN MATHEMATICS (STATISTICS), BACHELOR OF EDUCATION SCIENCE  
AND BACHELOR OF EDUCATION ARTS

AND

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN  
ACTUARIAL SCIENCE

### SMS 3355: DESIGN AND ANALYSIS OF SAMPLE SURVEYS

DATE: APRIL 2024

TIME: 2 HOURS

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INSTRUCTIONS: Answer Question ONE and any other TWO questions.

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#### QUESTION ONE (30 MARKS)

- a) Briefly explain the following as used in design and analysis of sample surveys; (6 Marks)
- analytical sample survey
  - parameter
  - efficient estimator
- b) For a simple random sample, the estimator for the population total  $X_T$ , is  $N\bar{x}$ . Find  $Var(N\bar{x})$  (4 Marks)
- c) Explain the two sources of errors in sample surveys. (3 Marks)
- d) Explain the uses of  $var(\bar{x})$  in estimation of population parameters. (4 Marks)
- e) As the estimator of  $\bar{X}$ , consider  $\bar{\bar{x}} = \frac{1}{n} \sum_{i=1}^n \bar{x}_j$ , the mean of means of the  $j^{th}$  clusters. Show that  $\bar{\bar{x}}$  is unbiased for  $\bar{X}$ . (5 Marks)



- f) Giving relevant examples, distinguish between quota sampling and stratified random sampling. (4 Marks)
- g) Consider a population of 5 units ( $N = 5$ ) with sampling units 0, 1,2,3 and 4. Show that for a simple random sample of size  $n = 3$ , the sample mean,  $\bar{x}$  is unbiased for the population mean  $\bar{X}$  (4 Marks)

**QUESTION TWO (20 MARKS)**

- a) Explain the two sampling schemes as used in design and analysis of sample surveys. (3 Marks)
- b) Given a random variable  $X$  is such that a simple random sample  $x_1, x_2, \dots, x_n$  of size  $n$  is selected, show that the variance of the simple random sample mean is given as  $var(\bar{x}) = \frac{S^2}{n}(1 - f)$  where  $S^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2$  and  $f = \frac{n}{N}$ . (8 Marks)
- c) Show that the variance of  $p$ , the sample proportion is given by  $var(p) = \frac{N-n}{N-1} \frac{PQ}{n}$  where  $P$  is the population proportion and  $Q = 1 - P$ . (9 Marks)

**QUESTION THREE (20 MARKS)**

- a) Justify the choice of stratified random samples in design and analysis of sample surveys. (3 Marks)
- b) Show that a stratified random sample mean,  $\bar{x}_{str}$  is more efficient than a simple random sample mean,  $\bar{x}$  as an estimator for the population mean,  $\bar{X}$ . (6 Marks)
- c) In a particular sector of industry, a survey is conducted to investigate the extent of absenteeism not connected with illness or official holidays. A random sample of 1000 men out of a total workforce of 36 000 are asked how many days they have taken off work, in the previous six months, as casual holidays. The results were as follows:

Days off	0	1	2	3	4	5	6	7	8	9
No. of men	450	163	188	111	49	20	6	10	2	1

- i. Obtain an estimate of  $X_T$  the total number of days taken off. (3 Marks)
- ii. Obtain an estimate of  $S^2$ . (3 Marks)



- iii. construct an approximate 90% symmetric two-sided confidence interval for  $X_T$   
(5 Marks)

**QUESTION FOUR (20 MARKS)**

- a) Giving examples, critically examine proportion estimators as used in survey sampling theory.  
(4 Marks)
- b) Show that the linear regression estimator, is at least as efficient as the ratio estimator, under all circumstances.  
(7 Marks)
- c) All the wheat farms in Nakuru county were stratified by farm size and the mean number of hectares of wheat per farm in each stratum, with the following results

Farm size (hectares)	No. of farms	Mean wheat (hectares)	Standard deviation
0-30	365	2.5	2.2
31-60	428	8.3	3.5
61-90	384	12.2	3.7
91-120	321	16.8	5.2
121-150	170	20.6	6.2
151-180	102	25.3	6.4
At least 180	138	31.9	9.2

For a sample of 100 farms, compute the sizes in each stratum under stratified random sampling with Neyman allocation.  
(9 Marks)

**QUESTION FIVE (20 MARKS)**

- a) Giving relevant examples, describe finite population correction (fpc) as used in sample surveys.  
(4 Marks)
- b) Given that  $\bar{y}_{sy}$  represents the estimator for the population mean  $\bar{Y}$  from the systematic sample, show that  $\bar{y}_{sy}$  is unbiased for  $\bar{Y}$   
(6 Marks)
- c) Let  $S_{wsy}^2 = \frac{1}{k(n-1)} \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{x}_i)^2$  and  $S^2 = \frac{1}{N-1} \sum_{i=1}^k \sum_{j=1}^n (x_{ij} - \bar{X})^2$  show that variance of the mean of a systematic sample is given by  $Var(\bar{x}_{sy}) = \frac{N-1}{N} S^2 - \frac{k(n-1)}{N} S_{wsy}^2$   
(10 Marks)

