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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

SMS 3459: DECISION THEORY AND BAYESIAN INFERENCE II

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Distinguish between classical statistical approach and Bayesian statistical approach (3makrs)

b) Define the following
 i. Conjugacy (2marks)
 ii. Kernel (2marks)
 iii. Credibility interval (2marks)
 iv. Jeffrey prior (2marks)

c) Let $y_1 \ldots\ldots x_n$ be exchangeable random quantities

i. State the 0-1 representation theorem (2marks)

ii. Show that the resulting distribution from 0-1 representation belong to the 1-paramenter exponential family, hence deduce the sufficient statistics for learning about $\emptyset$ (4marks)

d) For the following distributions, find the kernel, Jeffrey's prior and the corresponding distribution for θ
 i. $x_i/\theta \sim$poiss (θ) (5marks)
 ii. $x_i/\theta \sim$Bern(θ) (5marks)

e) Given $x_i/\theta \sim$poiss (θ) and $f(\theta) \sim$ gamma $(\propto, \beta)$ find the posterior distribution (3marks)

QUESTION TWO (20 MARKS)

a) Describe;

i. Metropolis Hasting algorithm (5marks)

ii. Importance sampling (5marks)

iii. Monte-carlo integration (5marks)

b) Let $x_i/\theta \sim$ Bino (n,p) and $f(\theta) \sim$ Beta $(\propto, \beta)$

i. Compute the posterior distribution (3marks)

ii. Derive the posterior mean and variance (2marks)

QUESTION THREE (20 MARKS)

a) Given $x_i/\theta \; poiss(\theta)$ and $\theta \sim$ gamma $(\propto, \beta)$ given the loss functor; $((\theta, d) = \theta(\theta - d)^2$. Find

i. The Bayes role and risk for immediate decision $(1B)$

ii. Bayes risk and role for sampling procedure for a sample size n (3marks)

iii. Bayes risk of the sampling procedure for a sample of site n (7marks)

QUESTION FOUR (20 MARKS)

a) Given $x_1 \dots . x_2$be exchangeable sequence such that $Y\!: |\theta \sim \mathrm{N}(\theta, \sigma^2)$ let on $\theta \sim \mathrm{N}(\mu_0, \sigma_0^2)$ let θ and σ^2 be unknown but $\mu_0, \sigma 0^2$ are known constants

i. Derive the posterior distribution (4marks)

ii. Determine the corresponding thernel, posterior mean and precision (6marks)

b) Suppose we want to sample $\theta|Y\!: \sim N(\theta, 1)$ using the metropolis hasting algorithm. Let the proposed distribution $q(\emptyset, \theta)$be the density of the $N(\theta, \sigma^2)$for same σ^2. Discuss the algorithm involved (6marks)

c) Discuss the following

i. Prior distribution (2marks)

ii. The parameter exponential family (2marks)

QUESTION FIVE(20MARKS)

Suppose we are interested in θ, the probability that a coin will yield a herd when a spun in a specified manner. We judge that the prior distribution is Beten $(4{,}4)$. The coin is spun 10 times, you did not witness. Rather than being told how many heads were seen less than three;

i. Find the posterior distribution up to proportionality and shown that the normaliting constant is given by $K = \frac{\sqrt{18}}{1536\sqrt{4}\sqrt{12}}$ (10marks)

ii. Show that the posterior mean is $\frac{39}{128}$ (10marks)