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UNIVERSITY EXAMINATIONS 2023/2024

THIRD YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3354: DECISION THEORY AND BAYESIAN INFERENCE I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define the following
   i. Statistical decision problem (2 Marks)
   ii. Admissibility (2 Marks)
   iii. Minimax (2 Marks)
   iv. Bayes risk (2 Marks)

b) Show that
   i. If a decision function δ^* is admissible with constant risk, then if is a minimax decision function. (4 Marks)
   ii. Bayes risk is always smaller than the minimax risk. (4 Marks)

c) Given the following pay off table

	Poor sales	Good sales
	E_1	E_2
New machine (d_1)	330	450
Overtime (d_k)	350	410
prob	0.35	0.65

i. Compute the EMV and determine the optimum decision. (4 Marks)

ii. Construct a decision tree for the problem. (4 Marks)

iii. How critical is the decision choice that probability for sales to be good is 0.65. (3 Marks)

iv. Compute the EMV under uncertainty (2 Marks)

v. Determine expected value of perfect information. (1 Mark)

QUESTION TWO (20 MARKS)

a) State the Bayes theorem. (3 Marks)

b) Given a coin was tossed n times, and heads observed. Consider a Beta Prior, deduce the pesters or distribution hence mean and variance. (6 Marks)

c) Let $x_1 \dots x_n$ be Bernoulli (p) consider two estimates $\tilde{p}_1 = \bar{y}(M.L.E)$ and $\hat{p}_2 = \frac{\sum_{1=i}^{n} x_i + \propto}{\propto + \beta + n}$

Using (Bayes estimator) the squared error loss. Compute

i. $RCp, \hat{p}_1$ (3 Marks)

ii. $RCp, \hat{p}_2$ (3 Marks)

d) Let $X \sim N(\theta, 1)$ consider two estimators $\tilde{\theta}_1 = 1$ and $\tilde{\theta}_2 = 3$, using the squared error loss compute the corresponding risk and determine the smallest risk. (5 Marks)

QUESTION THREE (20 MARKS)

a) On a given day the weather condition is either rainy $\tilde{\theta}_1$ or sunshine $\tilde{\theta}_2$. An individual has the following options; stay home (a_1) go out without umbrella (a_2) or go out with umbrella (a_3) as shown below.

Decision	θ_1Rainy	θ_2Sunny
a_1 (stay home)	4	5
a_2 go out without umbrella	5	2
a_3 go out with umbrella	3	0

b) Assume that probability that it rains is 0.5. compute the risk for all the decision and choose the optimum decision. (5 Marks)

c) Additional information is provided by the weather forecast $(y = y_1)$ with conditional probabilities as follows.

$p(y = y_1|\theta_1) = 0.6 \qquad p(y \neq y_1|\theta_2) = 0.8$
$p(y \neq y_1|\theta_1) = 0.4 \qquad p(y = y_1|\theta_2) = 0.2$

Compute the corresponding risks for each decision and choose the optimum decision. (5 Marks)

Calculate the Bayes risk and hence the prior value of information. (5 Marks)

QUESTION FOUR (20 MARKS)

A process provides certain item can be ok or flowed. let w denote the probability that a single item is ok. The production process can be in either of the two states; Runs well, or it run bad. If the process runs well, then 75% of the items produced are ok. If it run bad, then only 25% of the items produced ok. The chances that the process runs bad is 0.2. five items are sampled from the production line and investigated in order to determine if the process should be shot down or to continue running. The utility is given below.

	$w = {}^1/_4$	$w = {}^3/_4$
	Process runs bad	Process runs well
d_1	-10	-10
d_2	-30	0
prob	0.2	0.8

i. Compute loss function (4 Marks)

ii. Let x denote the number of Ok items, hence $x = K$ for $K = Q, 1,2,3,4 \; or \; 5 \; X \sim BIN(5, W)$ calculate all value of $P\left(X = K|w = {}^1/_4\right)$ and $P\left(X = k|w = {}^3/_4\right)$ (5 Marks)

iii. For each sample sES use Bayes theorem to compute the posterior probabilities $p(w|s)$ (3 Marks)

iv. Compute the risk with respect to d_1 and d_2. (4 Marks)

v. Obtain the values that both decision are Bayes. (2 Marks)

vi. Compute Bayes risk (2 Marks)

QUESTION FIVE (20 MARKS)

We need to decide to drill a particular well or not these are three possible states

State	Profit (utiles)	Probability
Dry (w_1)	-1	0.5
Wet (w_2)	1	0.25
Soaking(w_3)	5	0.25

We have possibility to pay an expert for inspecting the drill site, which costs 0.3 utiles. The expert opinion can be, sign are good (9) or signs are bad. (9 Marks)

From historical data about the expert opinion given all well types, we know that

Well type	G	$\bar{G}$
w_1	$p(G/w_1) = 0.2$	$p(\bar{G}/w_1) = 0.8$
w_2	$p(G/w_2) = 0.6$	$p(\bar{G}/w_2) = 0.4$
w_3	$p(G/w_3) = 0.8$	$p(\bar{G}/w_3) = 0.2$

Given the following pay off when expert advice is taken

	w_1	w_2	w_3
G	-1.3	0.7	4.7
G	-1.3	0.7	4.7

i. Draw a decision tree. (4 Marks)

ii. Should we pay for the expert opinion, should we drill. (10 Marks)

iii. Compute expected vale of information. (3 Marks)

iv. Compute the cost of taking the advice. (3 Marks)