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UNIVERSITY EXAMINATIONS 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMS 3465: CREDIBILITY THEORY AND LOSS MODELS

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) Differentiate between full and partial credibility [4 marks]

(b) An insurance company provides warranties for a certain electrical gadget. At the start of 2006 there were 4,500 gadgets under warranty, each of which has a probability q of suffering complete failure in 2006 (independently between gadgets). The prior distribution of q is beta with mean 0.015 and standard deviation 0.005. Given that 58 gadgets suffer a complete failure in 2006, determine the posterior distribution of q. [6 marks]

(c) An insurer's portfolio consists of three independent policies. Each policy can give rise to at most one claim per month, which occurs with probability θ independently from month to month. The prior distribution of θ is beta with parameters $\alpha = 2$ and $\beta = 4$. A total of 9 claims are observed on this portfolio over a 12 month period.

i. Derive the posterior distribution of θ. [4 marks]

ii. Derive the Bayesian estimate of θ under all or nothing loss. [4 marks]

(d) Describe the three types of credibility models. [6 marks]

(e) A coin is biased so that the probability of throwing a head is an unknown constant p. It is known that p must be either 0.4 or 0.75. Prior beliefs about p are given by the distribution: $P(p = 0.4) = 0.6$ $P(p = 0.75) = 0.4$ The coin is tossed 6 times and 4 heads are observed. Find the posterior distribution of p. [6 marks]

QUESTION TWO (20 MARKS)

An actuary has, for three years, recorded the volume of unsolicited advertising that he receives. He believes that the number of items that he receives follows a Poisson distribution with a mean which varies according to which quarter of the year it is. He has recorded Y_{ij} the number of items received in the ith quarter of the jth year ($i = 1,2,3,4$ and $j = 1,2,3$) The actuary wishes to estimate the number of items that he will receive in the 1st quarter of year 4. He has recorded the following data:

	Y_{i1}	Y_{i2}	Y_{i3}	$\overline{Y_i} = \frac{1}{3}\sum_j Y_{ij}$	$\sum_j (Y_{ij} - \overline{Y_I})^2$
$i = 1$	98	117	124	113	262
$i = 2$	82	102	95	93	206
$i = 3$	75	83	88	82	86
$i = 4$	132	152	148	144	224

i. Estimate $Y_{1,4}$ the number of items that the actuary expects to receive in the first quarter of year 4 using the assumptions of EBCT model 1. [6 marks]

The actuary believes that, in fact, the volume of items has been increasing at the rate of 10% per annum.

ii. Suggest how the approach in (i) can be adjusted to produce a revised estimate taking this growth into account. [3 marks]

iii. Calculate the maximum likelihood estimate of $Y_{1,4}$ (based on the quarter 1 data already observed and the 10% p.a. increase described above). [7 marks]

iv. Compare the assumptions underlying the approach in (i) and (ii) with those underlying the approach in (iii). [4 marks]

QUESTION THREE (20 MARKS)

Five years ago, an insurance company began to issue insurance policies covering medical expenses for dogs. The insurance company classifies dogs into three risk categories: large pedigree (category 1), small pedigree (category 2) and non-pedigree (category 3). The number of claims n_{ij} in the ith category in the jth year is assumed to have a Poisson distribution with unknown parameter θ_i. Data on the number of claims in each category over the last 5 years is set out as follows:

	Year						
	1	2	3	4	5	$\sum_{j=1}^{5} n_{ij}$	$\sum_{j=1}^{5} n_{ij}^2$
Category 1	30	43	49	58	60	240	12,144
Category 2	37	49	58	32	64	260	13,934
Category 3	26	31	18	37	32	144	4,354

Prior beliefs about θ_1 are given by a gamma distribution with mean 50 and variance 25.

(i.) Find the Bayes estimate of ϑl under quadratic loss. [6 marks]

(ii.) Calculate the expected claims for year 6 of each category under the assumptions of Empirical Bayes Credibility Theory Model 1 [7 marks]

(iii.) Explain the main differences between the approach in (i) and that in (ii). [4 marks]

(iv.) Explain why the assumption of a Poisson distribution with a constant parameter may not be appropriate and describe how each approach might be generalised. [3 marks]

QUESTION FOUR (20 MARKS)

The number of claims received by a motor insurance company on any given day follows a Poisson distribution with mean u. Prior beliefs about u are expressed through a gamma distribution with parameters a and b. Over a period of n days the observed number of claims received per day are $x_1, x_2 ..., x_n$.

a) Identify which one of the following is the posterior density of u: [4 marks]

$$f(u/x) \propto u^{b+\sum_{ij-1}} e^{-(a+n)u}$$

$f(u/x) \propto u^{b+\sum xi} e^{-(a+n+1)u}$

$f(u/x) \propto u^{a+\sum xi-1} e^{-(a+n)u}$

$f(u/x) \propto u^{b+\sum xj+1} e^{-(a+n-1)u}$

b) Write down the posterior density of the parameter u and specify its parameters. [3 marks]

c) (i.) Determine the Bayesian estimate of u under quadratic loss. [3 marks]

(ii) Write down the Bayesian estimate of u under quadratic loss as a credibility estimate and state the credibility factor. [3 marks]

Suppose that $a = 9, b = 3$ and that the company receives 320 claims in total during a 6-day period.

d) Calculate the Bayesian estimate of u under quadratic loss. [2 marks]

e) Calculate the variance of the posterior distribution of u. [2 marks]

An industry expert suggests that prior beliefs about u are better expressed through a gamma distribution with parameters $a = 18$ and$b = 6$.

f) Explain how these prior beliefs would affect the variance of the posterior distribution of u, without explicitly calculating the variance of the posterior distribution. [3 marks]

QUESTION FIVE (20 MARKS)

a. Let $x_1, x_2, ..., x_n$ independent observations from a Bernoulli distribution with $P(x_i = 1 = p, i = 1, ..., n.$ The parameter p has a beta prior distribution with parameters (a, b).

(i.) Determine the posterior distribution of parameter p. [6 marks]

(ii.) Determine the Bayesian estimate of parameter p under quadratic loss. [2 marks]

(iii.) Determine the Bayesian estimate of parameter p under quadratic loss as a credibility estimate, stating the credibility factor. [2 marks]

b. A statistician has recorded the number of advertising telephone calls that their office received over 2 years. The statistician has recorded data $X_{ij,}$ which is the number of calls received in the ith quarter of the jth year (where $i = 1,2,3$ and $j = 1,2$)

	X_{i1}	X_{i2}	$\overline{X_\iota}$	$\sum_{\forall j}(X_{ij-\overline{X_I}})^2$
$i = 1$	43	29	36	98
$i = 2$	38	42	40	8
$i = 3$	22	18	20	8
$i = 4$	68	356	62	72

(a.) Calculate values for:

i. $E[m(\theta)$ [1mark]

ii. $E[s^2(\theta)]$ [1marks]

iii. $var[m(\theta)]$ [2marks]

(b.) Calculate an estimate for X_{13}, the number of advertising telephone calls that the statistician's office expects to receive in the first quarter of year 3, using your answers to part (a) and the assumptions of the Empirical Bayes Credibility Theory Model 1 (EBCT Model 1). [1 mark]

(c.)

(i.) State two key assumptions underlying the EBCT Model 1. [2 marks]

(ii.) Explain what these assumptions mean for the data X_{ij} above. [2 marks]