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UNIVERSITY EXAMINATIONS 2023/2024

FIRST YEAR SECOND SEMESTER AND FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS

SMA 5106: COMPLEX ANALYSIS III

DATE: APRIL 2023 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Find the image of the region enclosed by the triangle $A(0,0), B(2,0), C(2,4)$ under the transformation $T(z) = z^2$ (7marks)

b) Show that the line joining $p = z + i$ and $Q = 1 + 2i$ is mapped onto the curve $V = \frac{1}{18}(81 - u^2)$ by the transformation $T(z) = z^2$ (7marks)

c) Show that $\sec^{-1} z = \frac{1}{i}\ln\left(\frac{1+\sqrt{1-z^2}}{z}\right)$ (6marks)

QUESTION TWO (20 MARKS)

a) Show that a composition of two linear fractional transformations is also a linear fractional transformation (7marks)

b) Determine a linear fractional transformation that maps $z = 1,0 - 1$ onto $w = i, \infty, 1$ respectively (4marks)

c) Find a linear fractional transformation which maps the upper half of the z-plane into the unit circle in the w-plane in such a way that $z = i$ is mapped to w=0, while points at infinity are mapped to w=-1 (5marks)

d) The transformation $w = \frac{i-z}{i+z}$ maps the upper half of the plane onto the unit circle. Show that a point z=x is mapped onto the point $w = \frac{1-x^2}{1+x^2} + i\frac{2x}{1+x^2}$ (4marks)

QUESTION THREE (20 MARKS)

a) Define a conformal mapping (3marks)

b) Discuss the conformality of $T(z) = z^2$ at the point (3,7) the intersection of the lines $l_1: x = 3$ and $l_2: y = x + 4$ (7marks)

c) State without proof the Argument theorem (3marks)

d) Use the Argument theorem to evaluate $\int \frac{f^{'/}(z)}{f(z)} dz$ when

i. $f(z) = z^6 - 4z^3 - z + 2$ and C encloses all zeros of $f(z)$ (3marks)

ii. $f(z) = \frac{(z^2+1)^2}{(z^2+2z+2)^3}$ and C is the circle $|z| = 4$ (4marks)

QUESTION FOUR (20 MARKS)

a) State and prove the maximum modulus theore (7marks)

b) State a Theorem called Rouches' theorem (3marks)

c) Determine the number of roots/zeros of $f(z) = 2z^5 - 6z^2 + z + 1$inside the region $1 \leq |z| \leq 2$ (5marks)

d) Show that the angle of rotation under the transformation $f(z) = (1 + i)z + (4 + i)$ is $^{\pi}/_{4}$. Determine the linear scale factor and the area scale factor (5marks)

QUESTION FIVE (20MARKS)

a) Explain the concept of analytic continuation (3marks)

b) Show that the series $f_1(z) = \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$ and $f_2(z) = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(2-i)^{n+1}}$ are analytical continuations of each other (5marks)

c) Define a Gamma function and prove the recursire property of the Gamma function i.e for $Re\ z > 0, \lceil(z + 1) = z\lceil(z)$ (5marks)

d) Given the$\Big\lceil(z)\Big\lceil(1 - z) = \frac{\pi}{\sin \pi z}$, show that $\Big\lceil(\frac{1}{2}) = \sqrt{\pi}$ and use the recursive property to show tha $\Big\lceil(\frac{9}{2}) = \frac{105\sqrt{\pi}}{16}$ (7marks)