MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya

Tel: +254(0) 799 529 958, +254(0) 799 529 959, + 254 (0) 712 524 293,

Website: info@must.ac.ke Email: info@must.ac.ke

University Examinations 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN MATHEMATICS AND BACHELOR OF SCIENCE IN EDUCATION

SMA 3453: COMPLEX ANALYSIS II

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Determine the residues of $f(z) = \frac{2z}{(z^2+1)(2z-1)}$ at each of its poles in the finite z-plane.

(6 marks)

b) Write $f(z) = \frac{3z+2}{z(z+1)(z-2)}$ as partial fractions, hence form a Laurent series expansion about $z = -1$ (8 marks)

c) Use residue theorem to evaluate the integral $\int_{-\infty}^{\infty} \frac{2x^2+3}{(x^2+9)^2} dx$ (8 marks)

d) Verify that the function $u(x, y) = x^3 - 3xy^2$ is harmonic hence find its conjugate $v(x, y)$

(8 marks)

QUESTION TWO (20 MARKS)

a) Show that the transformation $w = \left(\frac{2}{\sqrt{z}}\right) - 1$ transforms the region outside the parabola $y^2 = 4(1-x)$ into the interior of the unit circle in the w-plane $|w| = 1, let\ z = e^{i\theta}\ and\ w = e^{i\phi}$ (10 marks)

b) State the Schwarz reflection principle, hence test if the function f (z) = $4z^2$ - 3z + 2 satisfy the schwarz reflection principle (5 marks)

c) Prove $\Pi_n^{\infty}\left(1 - \frac{z^2}{n^2+1}\right)$ converges (5 marks)

QUESTION THREE (20 MARKS)

a) Distinguish between the following isolated singularities in terms of the Laurent series

i) Removable (1 mark)

ii) Essential (1 mark)

iii) Pole (1 mark)

b) Determine if the function $f(z) = 2xy + i(x^2 - y^2)$ satisfy the C-R equations (7 marks)

c) Compute $\int_0^{\infty} \frac{x^{\frac{1}{2}}}{1+x^2} dx$ using the residue theorem (l0 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate $\int_0^{2\pi} \frac{8}{5+2\cos\theta} d\theta\ for\ C : |z| = 1$, apply residue theorem (10 marks)

b) Determine a mobius transformation that maps the points $z_1 = 1, z_2 = 2i, z_3 = 4$ onto the points $w_1 = 1+i, w_2 = 3-i$ and infinity respectively (8 marks)

c) Differentiate between isogonal and conformal mapping (2 marks)

QUESTION FIVE (20 MARKS)

a) Find the residues of all singularities in $\mathbb{C}$ $of\ f(z) = \frac{z^3+5}{(\mathbb{Z}^{4-1})(\mathbb{Z}+1)}$ (10 marks)

b) If $g_1(z) = \int_0^{\infty} e^{-zt} dt\ and\ g_2(z) = i\sum_{n=0}^{\infty}\left(\frac{z+i}{i}\right)^n$, show that $g_1(z)$ and $g_2(z)$ are analytic continuation of each other in each case sketch the regions involved in the Argand diagram

(10 marks)