



# MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

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## University Examinations 2023/2024

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR  
OF SCIENCE IN EDUCATION

### SMA 3300: COMPLEX ANALYSIS I

DATE: APRIL 2024

TIME: 2 HOURS

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INSTRUCTIONS: Answer question *one* and any other *two* questions

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#### QUESTION ONE (30 MARKS)

a) If  $z_1 = 3 - i$  and  $z_2 = \sqrt{2} + 3i$  find

i.  $\bar{z}_1 z_2$  (2 marks)

ii.  $|z_1 - z_2|$  2 marks)

iii.  $\frac{z_1}{\bar{z}_2}$ , giving your answer in the form  $a + ib$  (3 marks)

b) Express  $z = -\sqrt{3} + 2i$  in polar form (3 marks)

c) Find the square roots of  $\sqrt{3} - 3i$  in the form  $a + ib$  (5 marks)

d) Find the modulus and conjugate of  $i^{17}$  (3 marks)

e) Identify the singularities, hence calculate the residues of the function

$$f(z) = \frac{e^{2z}}{z^2 - iz + 3z - 3i} \text{ at each singularity} \quad (5 \text{ marks})$$

f) Evaluate  $\lim_{z \rightarrow \infty} \frac{4z^2}{(z-1)^2}$  (4 marks)

g) Express  $\sinh(2+3i)$  in the form  $a+ib$  (3 marks)

**QUESTION TWO (20 MARKS)**

a) Calculate the radius and disc of convergence for the series  $\sum_{n=0}^{\infty} \frac{(z+i)^n}{(n-1)^3}$  (5 marks)

b) Find the principal value of  $(1-i)^{2-i}$  (5 marks)

c) Express  $z = -\sqrt{3} - i$  in trigonometric form (3 marks)

d) Evaluate the integral  $\oint_C |z|^2 dz$  around the square whose vertices are

$z_0 = 0, z_1 = 1, z_2 = 1+i$  and  $z_3 = i$  in a counter-clockwise sense (7 marks)

**QUESTION THREE (20 MARK)**

a) Show that the function  $u = e^x \sin y$  is harmonic, hence find its harmonic conjugate  $v(x, y)$

(6 marks)

b) Use the residue theorem (or otherwise) to evaluate the integral

i.  $\oint_C \frac{5z-2}{z(z-1)}, \quad C : |z|=2$  (4 marks)

ii.  $\int_0^{2\pi} \frac{d\theta}{4+3\sin \theta}$  (10 marks)

**QUESTION FOUR (20 MARKS)**

a) Use De Moiré’s theorem to prove the trigonometric identity  $\sin 2\theta = 2\sin \theta \cos \theta$

(4 marks)

b) Evaluate the integral  $\oint_C \frac{z+1}{z^4+2z^3} dz$ ,  $C:|z|=1$  using the Cauchy formula (4 marks)

c) Solve the equation  $z^5 = -1$  and sketch the roots geometrically (10 marks)

**QUESTION FIVE (20 MARKS)**

a) State the De Moivre's theorem (2 marks)

b) Simplify  $i^{800}$  (2 marks)

c) Evaluate the contours integral  $\int_C z^2 dz$  along the path from  $-1+i$  to  $4+3i$  comprising of

the line segments from  $-1+i$  to  $4+i$  and then from  $4+i$  to  $4+3i$  (9 marks)

d) Find a Laurent series expansion of the function  $f(z) = \frac{e^{2z}}{(z-1)^3}$  about  $z = 1$  (7 marks)