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University Examinations 2023/2024

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF DATA SCIENCE, BACHELOR OF SCIENCE COMPUTER SCIENCE AND BACHELOR OF COMPUTER TECHNOLOGY

SMA 3112: CALCULUS I

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Define the following terms as used in calculus

i. Domain (1 mark)

ii. Co-domain of a function (1 mark)

iii. Set (1 mark)

b) i. Given that $f(x) = x^3 + 2x^2 - 4x + 6$. Find the value of $f(2)$ (2 marks)

ii. Find the inverse of $f(x) = 2x - 1$ (2 marks)

c) Find the domain of the function $\frac{1}{x^2 - 9}$ (3 marks)

d) Given that $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$. Find the composite function fog (3 marks)

e) Evaluate the limits

i. $\lim_{x \to -2} \frac{3x^2 + x - 10}{x + 2}$ (3 marks)

ii. $\lim_{x \to \infty} \frac{x^2 + 3}{3x^2 + 2x + 1}$ (3 marks)

f) Use the first principles to find the derivative of the following $f(x) = \frac{1}{x}$ (3 marks)

g) Differentiate the following functions with respect to x

i. $g(x) = x^2 \sin x$ (3 marks)

ii. $f(x) = (2x + 10)^5$ (3 marks)

h) Compute $\frac{dy}{dx}$, given that $x = \cos t$ and $y = \sin t$ (2 marks)

QUESTION TWO (20 MARKS)

a) Evaluate

i. $\lim_{x \to 2} \frac{x^2 - 4}{x + 2}$ (2 marks)

ii. $\lim_{x \to 1} \frac{\left(1 - \sqrt{x}\right)}{(1 - x)}$ (3 marks)

iii. $\lim_{x \to \infty} \left(\frac{5x^2 - 3x + 2}{10x^2 - x + 100} \right)$ (2 marks)

iv. $\lim_{x \to 0} \left(\sqrt{1 + \frac{1}{x}} - \sqrt{\frac{1}{x}} \right)$ (3 marks)

b) Show that the function $f(x) = \left(\frac{x - 1}{x^2 + 2x + 2} \right)^{2/3}$ is continuous on the whole of real line

(3 marks)

c) Determine whether or not the function below is continuous at $x = 1$

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & if\ x \neq 1 \\ x = 1 \end{cases}$$ (3 marks)

d) Show that $f(x) = \dfrac{x^2 - 4}{x - 2}$ is not continuous at $x = 2$ (4 marks)

QUESTION THREE (20 MARKS)

a) Define the derivate of a function f at a number a (2 marks)

b) Use first principles to find the derivative of $g(x) = 3x^2$ (3 marks)

c) State the domain of $f(x) = \sqrt{x-1}$ (2 marks)

d) Use the definition of derivative in 3(a) above to find the derivative of $f(x) = x^2 - 4x + 3$ at a number a (5 marks)

e) Differentiate from first principles $g(x) = \dfrac{1-x}{2+x}$ (6 marks)

f) Find the derivative of $y = 4x^3 - 2x^2 + 7x + 3$ (2 marks)

QUESTION FOUR (20 MARKS)

a) Find the derivative of the following functions

i. $y = 12x^4 - 6x^3 + 2x^2 - 4x + 2$ (2 marks)

ii. $y = x^2 e^{2x}$ (2 marks)

b) Find the second derivative $\dfrac{d^2y}{dx^2}$ of $x = t^2, y = t^3$ (3 marks)

c) Find $\dfrac{dy}{dx}$ given that $y = xy^3 + x^2 y - 3x$ (4 marks)

d) Find the derivatives of the following functions

i. $y = e^{3x+2}$ (2 marks)

ii. $\dfrac{x^2 + 3x}{inx}$ (3 marks)

e) Find the derivative of the following functions

i. $y = x^2 + \cos x + e^x$ (2 marks)

ii. $y = 2\tan x - 5\sin x + In(x)$ (2 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the tangential velocity and acceleration of a particle when $t = 3$ given that $s(t) = t^3 + 2t^2 + 4t + 2$ (6 marks)

b) Classify the turning points of the function $y = (x-3)(2x+1)$ (6 marks)

c) A box of volume 32m3, having a square base and no top is to be considered from a material costing sh 80 per square meter. Find the dimensions of box such that the cost of material is minimum (8 marks)