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University Examinations 2023/2024

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE AND
BACHELOR OF SCIENCE IN MATHEMATICS

SMA 3457: ANALYTICAL APPLIED MATHEMATICS II

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions

QUESTION ONE (30 MARKS)

- a) Use a summation convention to write out explicitly the summation

$$y_i = a_{ij}x_j \text{ for } i, j = 1, 2, 3 \quad (4 \text{ marks})$$

- b) Find the inner product of the tensors A_{jk}^i and B_s^r (5 marks)

- c) Classify the integral equation $u(x) = x - \frac{x^3}{6} + \int_0^x (x-t)u(t)dt$ and state the Kernel and λ

value (3 marks)

- d) Use the generating functions for the Bessel functions to prove that $J_n(-x) = (-1)^n J_n(x)$

(6 marks)

e) Convert the integral equation $u(x) = x + \int_0^x (t-x)u(t)dt$ to an initial value problem and state the corresponding initial conditions (5 marks)

f) Prove the following recurrence relation for the modified Bessel functions

$$I'_n(x) = \frac{n}{x} I_n(x) + I_{n+1}(x) \quad (4 \text{ marks})$$

g) Prove that the contraction of the mixed tensor A^i_j is an invariant (3 marks)

QUESTION TWO (20 MARKS)

a) Prove the following recurrence relations for the modified Bessel functions

$$K'_n(x) = \frac{n}{x} K_n(x) - K_{n+1}(x) \quad (3 \text{ marks})$$

b) Find the equivalent Volterra integral equation to the following initial value problem

$$y''(x) + y(x) = \cos x, \quad y(0) = 0, \quad y'(0) = 1 \quad (7 \text{ marks})$$

c) Solve the integral equation $y(x) = 1 - \int_0^x (x-t)y(t)dt$ by the method of successive approximation (10 marks)

QUESTION THREE (20 MARKS)

a) Show that $u(x) = x$ is a solution of the Fredholm integro-differential equation

$$u'(x) = \frac{2}{3} + \int_0^1 tu(t)dt \quad (5 \text{ marks})$$

b) Prove that the general solution of the more general modified Bessel equation

$$x^2 y''(x) + xy'(x) + (\lambda^2 x^2 - n^2)y(x) = 0 \text{ is } AJ_n(\lambda x) + BY_n(\lambda x) \quad (6 \text{ marks})$$

c) Solve the integral equation $y(x) = e^{-x} + 2x + \int_0^x e^{t-x} y(t)dt$ by taking derivative with boundary condition $y(0) = 1$ (9 marks)

QUESTION FOUR (20 MARKS)

- a) Prove that K^{ij} is a second rank tensor, given that the double summation $K^{ij} A^i B^j$ is invariant for any two tensors A^i and B^j (5 marks)
- b) Prove the recurrence relation $\frac{d}{dx} \{x^{-n} I_n(x)\} = x^{-n} I_{n+1}(x)$ for the modified Bessel function (6 marks)
- c) Solve the following integral equation by Laplace transform method

$$y(t) = t^2 + \int_0^t y(u) \sin(t-u) du \quad (9 \text{ marks})$$

QUESTION FIVE (20 MARKS)

- a) Find the first covariant component of a tensor in cylindrical coordinates if its covariant coordinates in rectangular coordinates are $xy, 2y - z^2, xz$ (9 marks)
- b) Find the inverse Laplace transform $L^{-1} \left\{ \frac{1}{s(s^2 + 4)} \right\}$ using the convolution (6 marks)
- c) Show that the function $y(x) = \frac{1}{(1+x^2)^{3/2}}$ is a solution of the Volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t) dt \quad (5 \text{ marks})$$