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University Examinations 2023/2024

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE AND BACHELOR OF SCIENCE IN MATHEMATICS

SMA 3354: ANALYTICAL APPLIED MATHEMATICS I

DATE: APRIL 2024

TIME: 2 HOURS

INSTRUCTIONS: Answer question *one* and any other *two* questions

QUESTION ONE (30 MARKS)

a) Consider the differential equation $xy''(x) - \frac{y}{x+3} = 0$

- i. Find all the singularities of the given equation (2 marks)
- ii. Classify the singularities in (i) above (4 marks)

b) Express the improper integral $\int_0^{\infty} x^{\frac{1}{2}} e^{-\sqrt[3]{x}} dx$ as a Gamma function. Hence or otherwise

evaluate the integral (4 marks)

c) Use the definition of Bessel function to prove the recurrence relation

$$\frac{d}{dx} \{x^n J_n(x)\} = x^n J_{n-1}(x) \quad (5 \text{ marks})$$

d) Find the general solution of the partial differential equation $\frac{\partial^3 z}{\partial x \partial y^2} = \cos(x - 2y)$ (3 marks)

e) Find the inverse Laplace transform of the function

$$\frac{s-1}{s^2-6s+25} \quad (4 \text{ marks})$$

f) Solve the Lagrange auxiliaries $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{bx-cy}$ (5 marks)

g) Form a partial differential equation from the function $z = f(x^2 - y^2)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial y^2} + y^2 x^2 + \frac{3}{2} x^2 + 8x = f(y) \quad (2 \text{ marks})$$

b) Find the Laplace transform of the function $e^{-t} \sin 5t$ (4 marks)

c) Prove the recurrence formula $xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$ for Bessel functions (7 marks)

d) Find the inverse Laplace transform of the function $\frac{s}{(2s+1)(3s+1)}$ (7 marks)

QUESTION THREE (20 MARKS)

a) Find the Laplace transform of the function $t^3 \sin 2t$ (10 marks)

b) Solve the following initial value problem using Laplace transform

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 2y = 5 \sin t \text{ given that } y(0) = 0 \text{ and } y'(0) = 0 \quad (10 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) Express $\int_0^1 (1-x^3)^{-1/2} dx$ as a Beta function and leave your result in terms of the Gamma function (5 marks)
- b) Evaluate $\Gamma(-3/2)$ (3 marks)
- c) Obtain a Fourier series representation for the function $f(x) = x, -\pi < x < \pi$ (12 marks)

QUESTION FIVE (20 MARKS)

Find a power series solution for the differential equation $2xy''(x) - y'(x) + 2y = 0$ valid in the interval $|x| < R$