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University Examinations 2023/2024

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE MATHS/PHYSICS

SMA 3275: ALGEBRAIC STRUCTURES

DATE: APRIL 2024 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Let $f : A \to B$ be mapping from A from B. Define

i. An injective map (1 mark)

ii. A surjective map (1 mark)

b) Show that $a * b = 3 + ab \forall a, b \epsilon \mathbb{Z}$ is commutative but not associative (4 marks)

c) i. Prove that $G = (\mathbb{Z}_S^+ = \{1,2,3,4\} x_5)$ is a cylic group (3 marks)

ii. Determine the generator of the group G in (i) above (3 marks)

d) Given the set $S = \{(a, b): a, b \epsilon \mathbb{R}\}$ and define * by $(a,b)*(c,d) = ((a+bc), bd)$

i. Find the left identify of S (3 marks)

ii. Find the inverse of an element $(a, b) \epsilon S$ (5 marks)

QUESTION TWO (20 MARKS)

a) Define a subgroup (H,*) of a group (G,*) (2 marks)

b) Prove that:

i. The identify element of a subgroup in the same as that of the group (6 marks)

ii. The inverse of an element of a subgroup is the same as the inverse of the element regarded as the member of the group (6 marks)

c) i. Given the group $(\mathbb{Z}_6, +_6)$and its subgroup (H,$+_6$), where $H = \{1,3,5\}$. Determine the left cosets of $H\ in\ \mathbb{Z}_6$ (4 marks)

ii. Define the left coset of H in G (2 marks)

QUESTION THREE (20 MARKS)

a) Define the following terms

i. Binary operation * (2 marks)

ii. Closure property under a binary operation * (1 mark)

iii. Associative property of a binary operation * (1 mark)

iv. Existence of identity element (1 mark)

b) Given the set $T = \{(a, b): a, b \epsilon \mathbb{R}\}$ and a binary operation * defined on T by

$(a,b)*(c,d) = (ac - bd, ad + bc)$

i. T is closed under * (3 marks)

ii. * is commutative on T (3 marks)

iii. * is associative on T (3 marks)

iv. Does * admit identity element in T? (3 marks)

v. Does * admit inverse in T? (3 marks)

QUESTION FOUR (20 MARKS)

a) Define the following algebraic structures and give example of each

i. A commutative ring (4 marks)

ii. A field (4 marks)

iii. Subring (2 marks)

iv. Integral domain (2 marks)

b) Prove that a finite integral domain is a field (8 marks)

QUESTION FIVE (20 MARKS)

a) Show that the set G composed of f_1, f_2, f_3, f_4 4 transformations of the set of complex numbers in itself defined by $f_1(z) = z,\ f_2(z) = -z,\ f_3(z) = \frac{1}{z},\ f_4(z) = \frac{-1}{z}, \forall z \in \mathbb{C}$ is an abelian group with composite operation (11 marks)

b) Given that $f(x) = \frac{2x+7}{3}$ and $g(x) = 2x + 4$. Determine

i. $(gof)x)$ (2 marks)

ii. $(fog)^{-1}(-2)$ (4 marks)

c) Prove that every cyclic group is abelian (3 marks)