MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUER SCIENCE, APPLIED STATISTICS WITH PROGRAMMING AND MATHEMATICS AND ECONOMICS

AMS 449 - SURVIALANALYSIS

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question ONE and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define the term censoring and differentiate between right and left censoring. (3marks)

b) Define the following terms as used in survival analysis.

i. Probability density function. $f(t)$ (1mark)

ii. Survivalship function. $s(t)$ (1mark)

iii. Hazard function $h(t)$ (1mark)

c) (i)Estimate the survival curve for the following data using the Kaplan-Meier estimator.

$2,3^{+},2,9,16,18^{+},7,16,5,5^{+}$ (4marks)

(ii) Plot the survival curve in part (i) above and use it to estimate the median survival time for the data. (3marks)

d) A medical doctor treats 10 patients who visited his clinic and then tested them each day until the test was negative, the number of days of testing T was the random variable given in the table below. Determine S(t) and E(T) for the data below. (4marks)

Days (t)	1	2	3	4	5	6
No of patients tested that day	10	10	8	5	2	0

e) The exponential distribution of the form

$$f(t) = \begin{cases} \theta e^{-\theta t}, & t \geq 0 \\ 0, & elsewhere \end{cases}$$

is widely used by actuaries as a premise for the follow up studies. Derive the survival function and cumulated hazard function. (6marks)

f) If T has a weilbull distribution with α scale parameter and β as the index parameter. Show that T^{β} has an exponential distribution with parameter α^{β}. (5marks)

g) Explain TWO reasons why median is often used to describe the central tendency on survival distributions. (2marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Consider the survival data given in the table below.

Year of follow up	No alive at the beginning of interval	Number dying in interval
0-1	1100	240
1-2	860	180
2-3	680	184
3-4	496	138
4-5	358	118
5-6	240	60
6-7	180	52
7-8	128	44
8-9	84	32
≥9	52	28

Compute and plot the graphs of:

(i) Estimated survivorship function (4marks)

(ii) Probability density function. (4marks)

(iii) Hazard function. (4marks)

b) Consider a discrete random variable T for the future life time in days of a group of insects with the survival function given as.

t	0	1	2	3	4	5	6
S(t)	1	0.9	0.7	0.5	0.3	0.2	0

Find.

(i) F(t), the cumulative distribution function of T. (2marks)

(ii) P(T=t), the probability function of T. (2marks)

(iii) E(T) and SD(T) of T. (4marks)

QUESTION THREE (20MARKS)

a) The data given below shows a report of a clinical trial which was conducted to evaluate the efficiency of maintaining chemotherapy for acute leukaemia. The data given in weeks shows remission of patients who received the chemotherapy.

$2, 3^{+}, 6, 6, 7, 10^{+}, 15, 15, 16, 27, 30, 32.$

Calculate the Kaplan-Meier estimator and plot its graph. Estimate he median survival time from the graph (NB + denote censored observations). (10marks)

b) The following data gives 10 times to remission of Leukaemia patients receiving the drug 6mp have half patients receiving no patients (control groups). Times are in weeks and those censored are indicated with the plus sign.

Sample 1 receiving 6mp 25,18+,20+,22+,26+

Sample 0(Control) 17,20,21,21,22

Using Gehans generalize Wilcoxon test, to test whether the drug is effective at 5% level of significant (10marks)

QUESTION FOUR (20MARKS)

a) Show that the hazard function $h(t)$ is given by $h(t) = -\frac{d}{dt}\ln S(t)$ (4marks)

Where $S(t)$ is the survivor distribution

b) Define the following models.

(i) Proportional hazard model. (2marks)

(ii) Accelerated future time model. (2marks)

(iii) Show that under the Weilbull distribution the two models concides. (8marks)

c) The Weibull distribution is with two parameters $labda$ and k and it's a distribution for positive random variables with pdf given by

$$f(t) = \lambda k.(\lambda t)^{k-1}e^{-(\lambda t)^k}, \qquad t \geq 0$$

Calculate its CDF and survival function when $k = 1$ (4marks)