MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2024/2025 ACADEMIC YEAR

THIRD YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE ACTUARIAL SCIENCE

AMS 331 – PROBABILITY AND STATISTICS 1V

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question ONE and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Let X be a $3x1$ vector of random variable, X_1, X_2, X_3 variance-covariance matrix $\Sigma = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ and mean $\mu = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Consider the combination $Y_1 = x_1 + x_2$ and $Y_2 = x_1 + x_3 - 2x_2$. $Y^T = (y_1, y_2)$. Find the distribution of $\underline{Y}$. (5marks)

b) A bank teller serves customers standing in the queue one by one. Suppose that the service time x for customer i has mean $E(X_i) = 2$(minutes) and $Var(X_i) = 1$. We assume that service times for different bank customers are independent. Let Y be the total time the bank teller spends serving 50 customers. Find $p(90 < Y < 110)$. (4marks)

c) Define the following terms:

  i) Probability generating function (2marks)

  ii) Characteristics function (2marks)

d) Given that X and Y have a joint pdf

$$f(x,y) = \begin{cases} \frac{8}{9}(1+xy), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$$ calculate the

  i) Marginal pdf of X (3marks)

  ii) Marginal pdf of Y (3marks)

e) Obtain the general quadratic form of the matrix $D = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ (3marks)

f) A fair die is tossed 12 independent times. Determine the probability of the following configuration. (4marks)

Face	1	2	3	4	5	6
Number of occurrence	2	3	0	2	4	1

g) Examine if the weak law of large numbers hold for the sequence $\{x_p\}$of iid random variable with $p[x_k = (-1)^{k-1}.k] = \frac{6}{\pi^2 k^2}$ (4marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the characteristic function of binomial random variable (10marks)

b) Let $X_1, X_2, \ldots, X_{25}$ be iid with the following pmf

$$P_x(k) = \begin{cases} 0.6\,, & k = 1 \\ 0.4\,, & k = -1 \\ 0\,, & Otherwise \end{cases}$$

and let $Y = X_1 + X_2 + \ldots + X_n$. Using the CLT and continuity correction, estimate $P(4 \le Y \le 6$. (10marks)

QUESTION THREE (20 MARKS)

a) Two random variables X and Y have the joint pdf

$$f(x,y) = \begin{cases} kx, & 0 < y < x < 1 \\ 0, & elsewhere \end{cases}$$

i) Evaluate the constant k (3marks)

ii) Obtain marginal distribution of X and Y and show that the random variable are not independent. (7marks)

At a particular gas station, gasoline is stocked in bulk tank each week. Let random variable X denote the proportion of the tanks capacity that is stocked in a certain week and let Y denote the proportion of tanks capacity that is sold in the same week. Note that the gas station cannot sell more trin what was stocked in a given week. Which implies that the value of y cannot exceed the value of x. the possible joint pdf of X and Y is given by

$$f(x,y) = \begin{cases} 3x, & 0 \le y \le x \ge 1 \\ 0, & elsewhere \end{cases}$$

b)

i. Obtain the joint CDF of X and Y at the point $(x,y) = \left(\frac{1}{2}, \frac{1}{3}\right)$ (5marks)

ii. Find the probability that the amount of gas sold is less than half the amount that is stocked in a given week. That is, $P(Y < O.5X)$ (5marks)

QUESTION FOUR (20 MARKS)

a) Suppose that X has a poisson distribution with parameter λ. Obtain the pgf of X and hence find the mean and the variance of X. (10marks)

b) State and prove the weak law of large numbers. (5marks)

c) Given that $X \sim N(\mu, \Sigma)$

$$\Sigma = \begin{pmatrix} 100 & \frac{110}{3} & 20 \\ \frac{110}{3} & 121 & 44 \\ 20 & 44 & 64 \end{pmatrix} \quad , \ \mu = \begin{pmatrix} 33 \\ 44 \\ 27 \end{pmatrix}$$

Find the distribution of X_1 given that $X_2 = 39$ and $X_3 = 29$ (5marks)