



MURANG'A UNIVERSITY OF TECHNOLOGY
SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS AND ACTURRIAL
SCIENCE

UNIVERSITY POSTGRADUATE EXAMINATION

2024/2025 ACADEMIC YEAR

**FIRST YEAR FIRST SEMESTER EXAMINATION FOR MASTER OF
SCIENCE IN STATISTICS**

AMS 603 MULTIVARIATE METHODS

DURATION: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer any FOUR questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

QUESTION ONE (25 MARKS)

- a) Let $x_1, x_2, \dots, x_{20}$ be a random size of size $n=20$ from $N_6(\mu, \Sigma)$ population. Specify each of the following completely
- The distribution of $(X - \mu)' \Sigma^{-1} (X - \mu)$. (1 mark)
 - The distribution of $\bar{X}$ and $\sqrt{n}(\bar{X} - \mu)$. (2 marks)
 - The distribution of $(n-1)S$ (3 marks)
- b) Suppose $x_1, x_2, \dots, x_n$ are iid $N_p(0, \Sigma)$. Find the MIE of Σ . Show that it is biased estimators of Σ . (9 marks)
- c) Forty nine participants in study of a human aging were classified into diagnostic categories 'civilised' and 'not civilised' in the basis of an intensive psychiatrist examination. Determine the classifications criterion using the discriminate function for the results recorded in the table below. (10 marks)

	Group mean	
	No Civilisation	Civilisation
Information	12.57	8.75
Similarities	9.75	5.53
Arithmetic	11.49	8.50
Picture competition	7.97	4.75

$$S = \begin{pmatrix} 11.26 & 9.41 & 7.46 & 3.38 \\ & 13.53 & 7.38 & 2.50 \\ & & 11.58 & 2.62 \\ & & & 5.81 \end{pmatrix} \quad S^{-1} = \begin{pmatrix} -0.260 & -0.137 & -0.059 & -0.066 \\ & 0.187 & -0.038 & 0.016 \\ & & 0.151 & -0.017 \\ & & & 0.211 \end{pmatrix}$$

QUESTION TWO (25 MARKS)

- a) Let X be distributed as $N_3(\mu, \Sigma)$ where $\mu' = (1, -1, 2)$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$.

Which of the following random variable are independent? Explain.

- X_1 and X_2 (2 marks)
 - (X_1, X_3) and X_2 (3 marks)
 - X_1 and $X_1 + 3X_2 - 2X_3$ (3 marks)
 - Specify the conditional distribution of X_1 given that $X_2 = x_2$ and $X_3 = x_3$. (3 marks)
- b) i) Evaluate T^2 for testing $H_0 : \mu' = (7, 11)$, using the data

$$X = \begin{pmatrix} 2 & 12 \\ 8 & 9 \\ 6 & 9 \\ 8 & 10 \end{pmatrix} \quad (4 \text{ marks})$$

- Specify the distribution of T^2 for the situation in (i) (2 marks)
- Using (i) and (ii) test H_0 at the $\alpha = 0.05$. What conclusion do you reach? (3 marks)

QUESTION THREE (25 MARKS)

Observations on two responses are collected for three treatments. The observation (x1) vector

$$\text{Treatment 1 } \begin{pmatrix} 6 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 9 \end{pmatrix}, \begin{pmatrix} 8 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 9 \end{pmatrix}, \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

$$\text{Treatment 2 } \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\text{Treatment 3 } \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

- Break up the observations into mean, treatments and residual components. Construct the corresponding arrays for each variable. (6 marks)
- Using the information part (i) construct the one-way MANOVA table. (14 marks)
- Conduct a test of hypothesis using $\alpha = 0.05$ the test equally of the treatment means. (5 marks)

QUESTION FOUR (25 MARKS)

- Let X be of dimension $n \times p$ with rank p . Show that $e = My$ and $b = Hy$ are independently distributed where $y \sim N(X\beta, \sigma^2 I)$ and $H = X(X'X)^{-1}X'$ is $n \times n$ and $M = I - X(X'X)^{-1}X'$. (8 marks)
- Consider the annual rates of returns (including dividends) on the Don-Jones industrial average for the years 1996-2005. The data multiplied by 100, are 0.6, 3.1, 25.3, 16.8, 7.1, 6.2, 25.2, 22.6, 26.0. Using these nine observations to complete the following:
 - Construct a Q-Q plot. Do the data seem to be normally distributed? Explain. (8 marks)
 - Carry out test of normality based on the correlation coefficient. Let the $\alpha = 0.01$. (9 marks)

QUESTION FIVE (25 MARKS)

- What is principal components analysis? (2 marks)
- State two uses of principal components analysis (2 marks)
- The data for the variable X_1, X_2 and X_3 were collected and recorded below

X_1	7	5	4	4	5
X_2	4	6	4	4	2
X_3	2	1	2	2	3

Find the principal components loading, scores and interpret your findings. (21 marks)