MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY ORDINARY EXAMINATION

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

AMS 412 – FURTHER DISTRIBUTION THEORY

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question ONE and any other two questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) State the central limit theorem. (3marks)

b) Let $Y = X_1 + X_2 + ... + X_{20}$ be the sum of random samples of size 20 from the distribution whose density function is given by $f(x) = \begin{cases} \frac{3}{2}x^2 & -1 < x < 1 \\ 0 & otherwise \end{cases}$

What is the approximate value of $P(-0.3 < x < 1.5)$ when one uses the central limit theorem? (10 marks)

c) Show the probability mass function of the multinomial distribution, its assumptions and moments. (4marks)

d) Derive the moment generating function of the multinomial distribution. (5marks)

e) Suppose that the racial/ethnic distribution in a large city is given by the table below. Consider these three options as the parameters of a multinomial distribution

Black	Hispanic	Other
20%	15%	65%

Suppose that a jury of 12 members is chosen. Find the probability that the jury contains;

i) Three black and eight other members (3marks)

ii) Four black eight other members (3marks)

iii) At most one black member (2marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Let $X_1 < X_2 < .... < X_n$ be a random sample of size n from a distribution with density function $f(x)$. Them the probability density function of the j^{th} order statistics $X_{(j)}$

$g(\mathrm{x}_{(j)}) = \frac{n!}{(j-1)!(\mathrm{n}-\mathrm{j})!}\left[F(x)\right]^{j-1}\left[F(x)\right]^{n-j} f(x)$. Prove. (10marks)

b) Let $Y_1 < Y_2 < .... < Y_n$ be it order statistics of a random sample of size six from distribution with a probability density function $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases}$

Find:

i) The joint probability function of $y_1,\ y_2,\ ...,\ y_5$. (2marks)

ii) The 4^{th} order statistics (4marks)

iii) Joint pdf of 2^{nd} and 4^{th} order statistics (4marks)

QUESTION THREE (20 MARKS)

a) If a symmetric matrix and y is a vector, express them in a quadratic form. (2marks)

b) State the weak law of large numbers (3marks)

c) State the strong law of large number (3marks)

d) Let $x_1, x_2, ..., x_n$ be iid Bernoulli random variables with parameter p. verify the law of large numbers. (3 marks)

e) Let $x_1, x_2, ..., x_n$ be iid from the normal distribution random variable with mean μ and variance σ^2. Find the distribution of sample variance S^2, its mean and variance. (9 marks)

QUESTION FOUR (20 MARKS)

a) Let X and Y be independent random variables, each with a density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad -\infty < x < \infty$. Let $U = XY$ and $V = Y$.

   i) What is the joint density of U and V? (9marks)

   ii) What is the density of U? (5marks)

b) Let $X \sim P(\lambda_1)$-and $Y \sim P(\lambda_2)$. What is the probability density function of $X + Y$ if X and Y are independent? (6marks)