MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY ORDINARY EXAMINATION

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMS 405– DESIGN AND ANALYSIS OF EXPERIMENTS II

DURATION: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer question ONE and any other two questions.

2. Mobile phones are not allowed in the examination room.

3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

A company markets its products by direct mail. An experiment was done to study the effects of three factors on the response rate to a product. The three factors were A- type of mail (3rd class, 1st class), B= brochure type (colour, black and white) and C= offered price (KES 1995, KES 2495). The response variable is the number of orders placed. The response variable is the number of orders place. The experimental data are as shown below.

Coded Factors			Number of Orders	
A	B	C	Replicate 1	Replicate 2
-	-	-	50	54
+	-	-	44	42
-	+	-	46	48
+	+	-	42	43
-	-	+	49	46
+	-	+	48	45
-	+	+	47	48
+	+	+	56	54

a) Estimate the factor effects.   (10 marks)

b) Prepare an ANOVA table and determine which factors are important in explaining the number of orders placed.   (10 marks)

c) Write R code that can be used to perform this analysis.   (2 marks)

d) Suppose that only one half fraction of the design could be run. Set up the design with I=ABC and analyse the data.   (8 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) An experiment was performed to investigate the capability of a measurement system. Five parts were randomly selected and two randomly selected operators measured each part three times. The tests were done in random order and the data are shown

Part Number	Operator 1 Measurements			Operator 2 Measurements		
	1	2	3	1	2	3
1	50	49	50	50	48	51
2	52	52	51	51	51	51
3	53	50	50	54	52	51
4	49	51	50	48	50	51
5	48	49	46	48	49	48

(i) Analyse the data from this experiment. (7 marks)

(ii) Estimate the variance components using the ANOVA method. (3 marks)

b) Does a new exercise program improve endurance over time? Five participants were measure on their endurance at three time points; before the program (pre-test) after four weeks (post-test) and after 8 weeks (follow up). The number of minutes one can run on a tread mill without stopping formed an endurance measure resulting in the following data:

Participant	Pre-test	Post test	Follow up
1	15	20	25
2	10	12	14
3	18	22	24
4	12	19	17
5	20	25	30

Analyse the data and draw conclusions. Use $\alpha = 0.05$ (10 marks)

QUESTION THREE (20 MARKS)

The percentage of hardwood concentration in raw pulp, pressure and cooking time of pulp are being investigated for their effects on the strength of hyper. A factorial experiment with two replicates is conducted and the following data are obtained:

Hard wood Concentration	Cooking time = 3 hours Pressure			Cooking time = 4 hours Pressure		
	400	500	650	400	500	650
2	6.6	7.7	9.8	8.4	9.6	10.6

	6.0	6.0	9.4	8.6	10.4	10.9
4	8.5	6.0	8.4	7.5	8.7	9.6
	7.2	6.9	7.6	8.1	8.0	9.0
8	7.5	5.6	7.4	7.6	7.0	8.5
	6.6	6.2	8.1	8.4	7.8	9.8

Analyse the data and draw conclusions. Use $\alpha = 0.05$ (20 marks)

QUESTION FOUR (20 MARKS)

The hexagon design below is used in an experiment that has the objective of fitting a second order response model

x_1	x_2	y
1.0	0.0	68
0.5	$\sqrt{0.75}$	74
-0.5	$\sqrt{0.75}$	65
-1.0	0.0	60
-0.5	$-\sqrt{0.75}$	63
0.5	$-\sqrt{0.75}$	70
0.0	0.0	58
0.0	0.0	60
0.0	0.0	57
0.0	0.0	55
0.0	0.0	69

a) Fit a second order model given that

$$(X'X)^{-1} = \frac{1}{30}\begin{bmatrix} x_0 & x_1 & x_2 & x_1^2 & x_1x_2 & x_2^2 \\ 6 & 0 & 0 & -6 & 0 & -6 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ -6 & 0 & 0 & 21 & 0 & 1 \\ 0 & 0 & 0 & 0 & 40 & 0 \\ -6 & 0 & 0 & 1 & 0 & 21 \end{bmatrix}$$

b) Perform canonical analysis. What type of surface has been found?   (5 marks)

c) What operating conditions on x_1 and x_2 lead to the stationary point?   (5 marks)