MURANG'A UNIVERSITY OF TECHNOLOGY

SCHOOL OF PURE, APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY POSTGRADUATE EXAMINATION

2024/2025ACADEMIC YEAR

FIRST YEAR **FIRST** SEMESTER EXAMINATION FOR DOCTOR OF PHILOSOPHY IN STATISTICS

AMS- 801- ADVANCED STISTICAL INFERENCE

DURATION:3 HOURS

INSTRUCTIONS TO CANDIDATES:

1. Answer any FOUR questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

QUESTION ONE (25 MARKS)

a) Discuss three limits theorems and their implications. (4 marks)

b) Let the probability density function of a random variable

$$f(x)=\begin{cases}630x^4(1-x)^4 & 0<x<1\\ 0 & otherwise\end{cases} \text{ i)}$$

i) What is the exact value of $P(|X-\mu|\le 2\sigma)$? (7 marks)

ii) What is the approximate value of $P(|X-\mu|\le 2\sigma)$ when one uses the Chebyshev inequality?

(2 marks)

c) Let $Y = X_1 + X_2 + ... + X_{15}$ be the sum of a random sample of size 15 from the distribution whose density function is $f(x)=\begin{cases}\frac{3}{2}x^5 & -1<x<1\\ 0 & otherwise\end{cases}$

What is the approximate value of $P(-0.3 < X < 1.5)$ when one uses the central limit theorem?

(7 marks)

QUESTION TWO (25 MARKS)

a) Differentiate between the following terms:

i) Prior and posterior distribution (2marks)

ii) Non – information and formative prior. (2 marks)

b) Suppose that X is a random variable with mean μ and variance σ^2 where σ^2 is known and μ is unknown. Suppose that μ behaves as a random variable whose probability distribution (prior) $\pi(\mu)$ and is also normally distributed with mean μ_p and variance of σ_p^2 both assumed to be known or estimated. Find the posterior distribution of $f(\mu | x)$. (13 marks)

c) A student taking a standardized test is classified as gifted if he or she scores as at least 100 out of a possible score of 150. Otherwise the student is classified as not gifted. Suppose the prior distribution of the scores if all students is a normal with mean 100 and standard deviation 15. It is believed that scores will vary each time the student takes the test and that scores can be modeled as a normal distribution with mean u and variance 100. Suppose the student takes the test and scores 115. Test the hypotheses that the student can be classified as a gifted student. (8 marks)

QUESTION THREE (25 MARKS)

a) Describe two resampling methods. (6 marks)

b) The following data represent the total ozone levels measured in Dobson units at randomly selected locations on earth on a particular day.

269, 246, 388, 354, 266, 303

i. Find a Jack-knife point estimate of the population mean m ozone level. (6marks)

ii. Construct a 95% Jackie confidence interval for the population mean u. (5 marks)

c) The following data represent the total ozone levels measured in Doson units at randomly selected location on earth on a particular day: 209,246,388,388,354,266,303

Suppose two Bootstrap sample of size five and draw from the measurements. Assume this random samples are 388, 209 246,354,266 and 266,246,388 354, and 303. Find the bootstrap mean, standard deviation, and standard error of the mean. (8 marks)

QUESTION FOUR (25 MARKS)

a) What is the difference between parametric and non-parametric methods? (2 marks)
b) Give an example of a non-parametric test equivalent to ANOVA. (1 mark)
c) Give an example of a non-parametric test equivalent to paired t-test. (1 mark)
d) What is a robust estimator? (3 marks)
e) Give two examples of robust estimators. (2 marks)
f) Discuss the general approaches to robust estimation. (4 marks)
g) Describe the difference between a linear and quartile regression. (4 marks)
h) Let $\{x_i\}\ i=1,2,...,N$ be points in $\mathbb{R}^k$ with the outputs $\{y_i\}\ i=1,2,...,N$. Let $X=\{x_1,x_2,...,x_N\}$ we define the regression quartile as $\hat{\beta}_r=\arg\min_{\beta\in\mathbb{R}^k}\sum_i^N \rho_r\{y_i-x'\beta\}$. Prove that the solution of the problem is equivalent to the solution of the following linear equation. $\arg\min_{\beta\in\mathbb{R}^k,U,V\in\mathbb{R}^N}(U'\tau+V'(1-\tau))$ subject to $X'\beta-y+u-v=0$, $u,v\geq 0$. (8 marks)

QUESTION FIVE (25 MARKS)

a) Define the following terms and given example in each:
   i) Asymptotically unbiased estimator. (3 marks)
   ii) Asymptotically relative efficiency. (3 marks)

b) Let $x_1,x_2,...,x_n$ n>3 be a random sample forms population with a true mean μ and variance σ^2. Consider the following three estimators of μ.

$$\hat{\theta}_1=\frac{1}{3}(X_1+X_2+X_3)$$

$$\hat{\theta}_2=\frac{1}{8}X_1+\frac{3}{4(\text{n}-2)}(X_2+...+X_{n-1})+\frac{1}{8}X_n$$

$$\hat{\theta}_3=\overline{X}$$

i. Show that each of these estimators is unbiased. (6 marks)
ii. Find relative efficiency of $e(\hat{\theta}_2,\hat{\theta}_1)$, $e(\hat{\theta}_3,\hat{\theta}_1)$ and $e(\hat{\theta}_3,\hat{\theta}_2)$. (11 marks)
iii. Which of the three estimates is the most efficient? (2 marks)